GCE A Level Maths 9709

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1.1 Quadratics

In this topic we will learn how to:

- solve quadratic equations, by factorising, completing the square and using the formula

Solving Quadratic Equations

There are three ways of solving quadratics:

1. Factorisation

   A quadratic equation can be solved by sight using factorization.

   (a) When the coefficient of x^2 is 1

   To factorise the quadratic x^2+bx+c , find the pair of factors of c that add up to b. Let's take a look at the example below.

   Solve $x^2+5x-6=0$ by factorization.

   $$x^2+5x-6=0$$

   The first step is to open two sets of parentheses next to each other each with an x in them and equate them to 0,

   $$(x\quad)(x\quad)=0$$

   Identify b and c

   $$b=5,\ c=-6$$

   Find pair factors of c,

   $$-6 \text{ and } 1;\ 6 \text{ and } -1;\ 3 \text{ and } -2;\ -2 \text{ and } 3$$

Find the pair of factors that add up to b,

$$6 \text{ and } -1$$

Add each number to one of the two parentheses we opened earlier,

$$(x+6)(x-1)=0$$

Note: If you were to expand the two sets of parentheses you should get the original equation.

Equate each bracket to 0**,**

$$x+6=0 \qquad\qquad x-1=0$$

Solve the two linear equations,

$$x=-6 \qquad\qquad x=1$$

Therefore, your roots are:

$$x=-6, \qquad x=1$$

(b) When the coefficient of x^2 is not 1

Factorizing a quadratic equation when the coefficient of x^2 is not 1 is a bit more challenging, however, with enough practice, it will become easier. Let's take a look at the example below.

Solve $2x^2+9x+10=0$ by factorization.

$$2x^2+9x+10=0$$

The first step is to identify a, b and c,

$$a=2, \qquad b=9, \qquad c=10$$

Then find the product of a and c,

$$2 \times 10 = 20$$

List the pair factors of 20**,**

20 and 1; 10 and 2; 5 and 4; -20 and -1; -10 and -2; -5 and -4;

Find the pair of factors that add up to b,

$$5 \text{ and } 4;$$

Rewrite b as the sum of these factors,

$$2x^2 + 9x + 10 = 0$$

$$2x^2 + (5 + 4)x + 10 = 0$$

Remove the parentheses,

$$2x^2 + 5x + 4x + 10 = 0$$

Now we will factorize by grouping. Group terms that are multiples of each other,

$$(2x^2 + 4x) + (5x + 10) = 0$$

Factorise the expressions in parentheses,

$$2x(x + 2) + 5(x + 2) = 0$$

We then factor out the $(x + 2)$ since it is common,

$$(2x + 5)(x + 2) = 0$$

Equate each bracket to 0,

$$2x + 5 = 0 \qquad x + 2 = 0$$

Solve the two linear equations,

$$x = -\frac{5}{2} \qquad x = -2$$

Therefore, your roots are:

$$x = -\frac{5}{2}, \qquad x = -2$$

Note: With practice, most of the steps outlined above will become intuitive and you can skip them. Factorising by sight should be the method you use in solving a quadratic equation unless told otherwise.

2. Completing the square

To solve a quadratic equation using the completing the square method you first have to complete the square. After completing the square, make x the subject of the formula. Let's take a look at an example below.

Solve $x^2 + 5x - 6 = 0$ by first completing the square.

$$x^2 + 5x - 6 = 0$$

The first step is to complete the square,

$$\left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 6 = 0$$

Simplify,

$$\left(x + \frac{5}{2}\right)^2 - \frac{49}{4} = 0$$

Once you have completed the square you have to make x the subject of the formula. To do that we will start by moving the term outside the parentheses to the other side of the equal sign,

$$\left(x + \frac{5}{2}\right)^2 = \frac{49}{4}$$

Take the square root of both sides to get rid of the power 2,

$$\sqrt{\left(x + \frac{5}{2}\right)^2} = \pm\sqrt{\frac{49}{4}}$$

Note: We put a $\pm$ sign, whenever we take the square root of a number.

The square root gets rid of the power 2 on the left hand side,

$$x + \frac{5}{2} = \pm\frac{7}{2}$$

Make x the subject of the formula,

$$x = -\frac{5}{2} \pm \frac{7}{2}$$

Since there is a $\pm$ sign, we can split the equation above into two separate equations,

$$x = -\frac{5}{2} + \frac{7}{2} \qquad x = -\frac{5}{2} - \frac{7}{2}$$

So our equation has two solutions which are,

$$x = 1 \qquad x = -6$$

3. Quadratic Formula

For the quadratic equation $ax^2+bx+c = 0$. The quadratic formula is,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

,

This can be used to solve any quadratic equation, where a, b and c are known constants.

Let's take a look at an example below.

Solve $x^2 + 5x - 6 = 0$ using the quadratic formula.

The first step is to identify the values of a, b, and c,

$$a = 1,\ b = 5,\ c = -6$$

Substitute $a = 1$, $b = 5$, $c = -6$ into the quadratic formula,

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2(1)}$$

Simplify,

$$x = \frac{-5 \pm \sqrt{25 + 24}}{2}$$

$$x = \frac{-5 \pm \sqrt{49}}{2}$$

$$x = \frac{-5 \pm 7}{2}$$

The above can be written as two separate equations,

$$x = -\frac{5}{2} + \frac{7}{2} \qquad x = -\frac{5}{2} - \frac{7}{2}$$

So our final solutions are,

$$x = 1 \qquad x = -6$$