GCE A Level Maths 9709

SMIYL

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1.2 Functions

In this topic we will learn how to:

- understand and use transformations of the graph of $y = f(x)$ given by, $y = f(x)+a$, $y = f(x+a)$, $y = af(x)$, $y = f(ax)$ and simple combinations of these.

Transformations

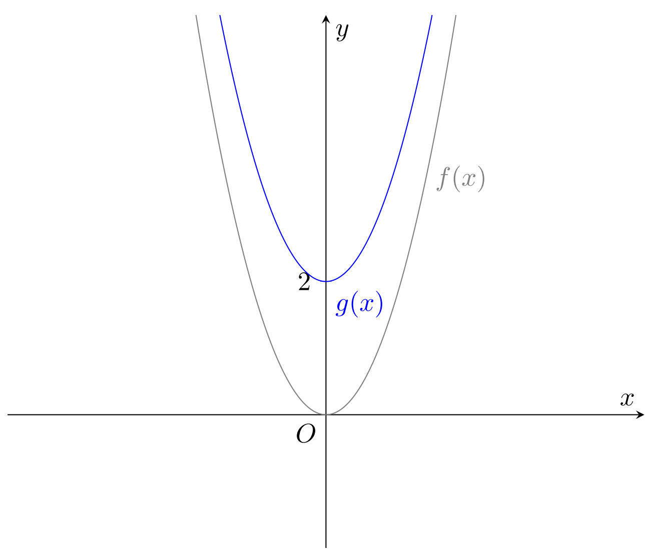
y = f(x) + a

This is a translation in the y-axis by a units.

Here's an example of what that would look like graphically.

Example 1

Given that $f(x) = x^2$ sketch the graph of $g(x) = f(x) + 2$.



$g(x)$ **is a translation by** 2 **units in the** y**-direction from** $f(x)$

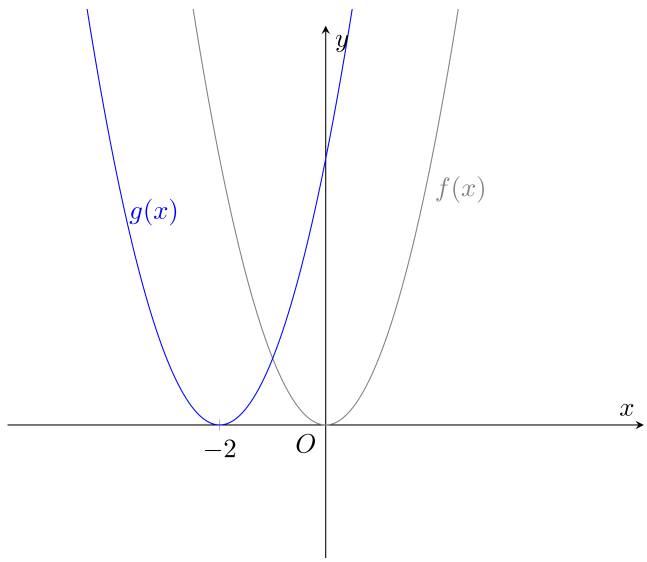
$$\mathbf{y = f(x + a)}$$

This is a translation in the x-axis by $-a$ **units.**

Here's an example of what that would look like graphically.

Example 2

Given that $f(x) = x^2$ sketch the graph of $g(x) = f(x + 2)$.



$g(x)$ **is a translation by** -2 **units in the** x**-direction from** $f(x)$

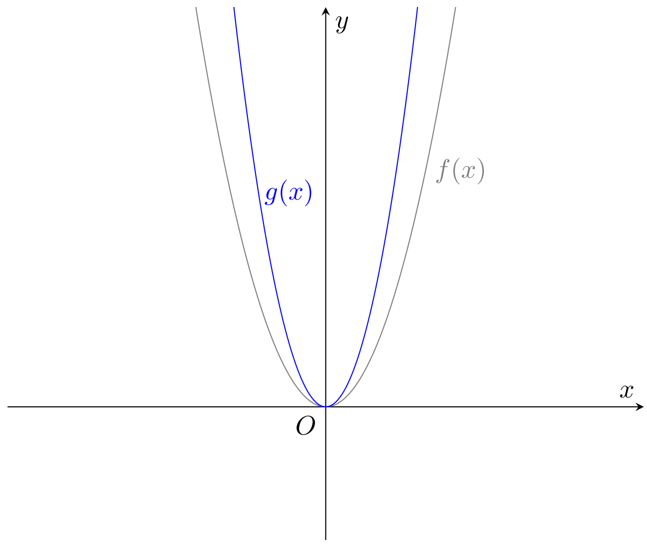
$$\mathbf{y = af(x)}$$

This is a stretch in the y-axis by a stretch factor of a**.**

Here's an example of what that would look like graphically.

Example 3

Given that $f(x) = x^2$ sketch the graph $g(x) = 2f(x)$.



$g(x)$ is a stretch in the y-direction by a stretch factor of 2 from $f(x)$

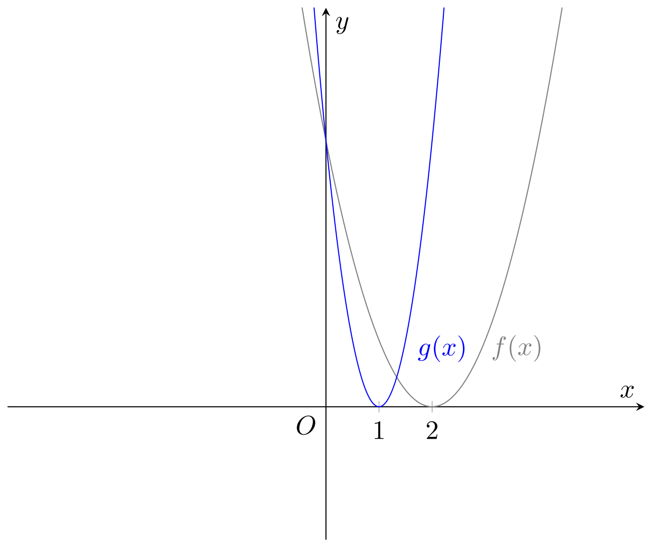
$$\mathbf{y = f(ax)}$$

This is a stretch in the x-axis by a stretch factor of $\frac{1}{a}$.

Here's an example of what that would look like graphically.

Example 4

Given that $f(x) = (x-2)^2$ sketch the graph $g(x) = f(2x)$



$g(x)$ is a stretch in the x-direction by a stretch factor of $\frac{1}{2}$ from $f(x)$

Combined Transformations

$$a(bx+c)^2+d$$

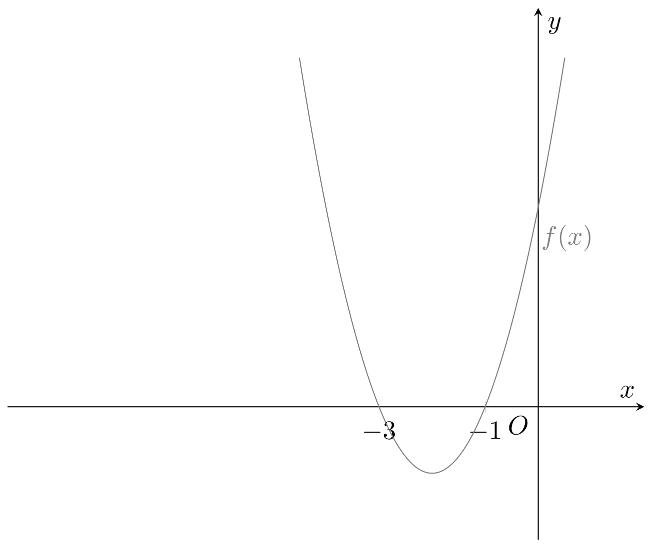
The above is an example of a combined transformation. a represents a stretch in the y-axis. b represents a stretch in the x-axis. c represents a translation in the x-axis. d represents a translation in the y-axis.

Here's an example of a combined transformation.

Example 5

Given that $f(x) = (x+2)^2 - 1$ sketch the graph $g(x) = 3f(2x+4) + 1$.

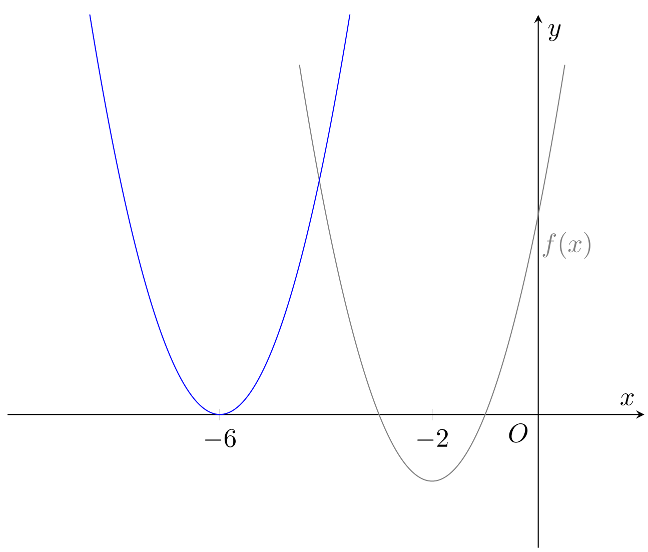
Start by sketching the graph of $y = f(x)$,



$$y = f(x)$$

$$g(x) = 3f(2x + 4) + 1$$

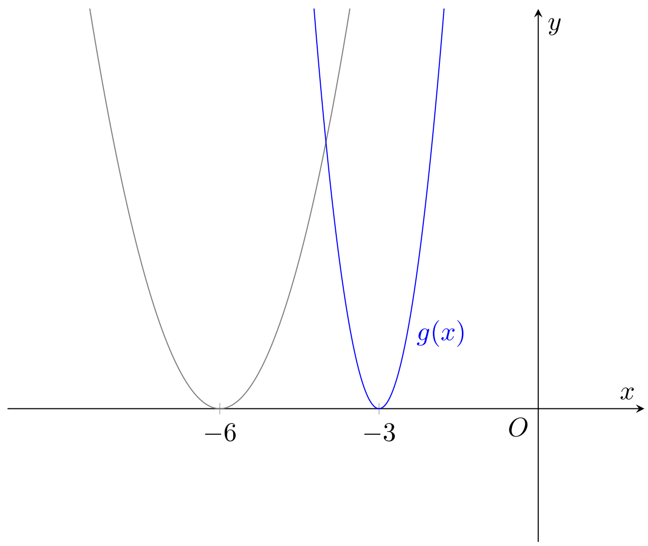
From $f(x)$ to $g(x)$ there is a translation in the x-direction by -4 units and a translation in the y-direction by 1 unit. Let's translate $f(x)$ by -4 units in the x-direction and 1 unit in the y-direction,



Translation by -4 units in the x-axis and 1 unit in the y-direction from $f(x)$

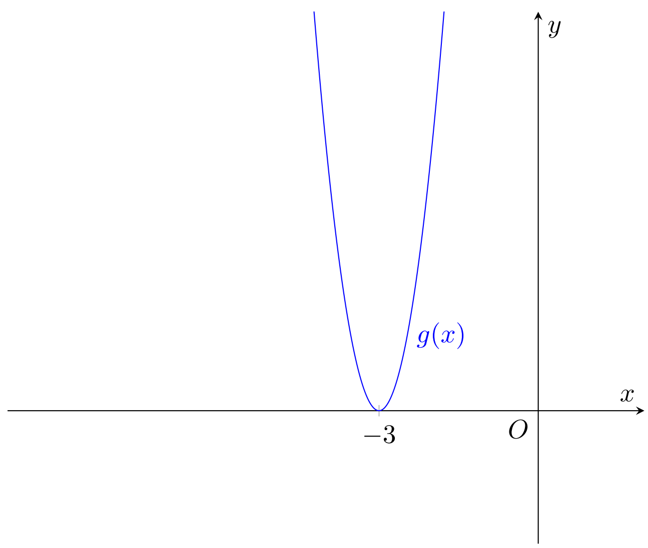
$$g(x) = 3f(2x + 4) + 1$$

Finally, there is a stretch in the x-direction by a stretch factor of $\frac{1}{2}$ and a stretch in the y-direction by a stretch factor of 3,



Stretch in the x-direction by a stretch factor of $\frac{1}{2}$ and stretch in the y-direction by a stretch factor of 3

Therefore, the graph of $y = g(x)$ is,



$y = g(x)$

Let's look at some past paper questions.

1. The graph of $y = f(x)$ is transformed to the graph $y = 1 + f\left(\frac{1}{2}x\right)$. Describe fully the transformations which have been combined to give the resulting transformation. (9709/12/F/M/20 number 2)

   Start by describing the transformation in the x-axis,

   There is a stretch in the x-direction by a stretch factor of 2.

   Then describe the transformation in the y-axis,

   There is a translation in the y-direction by 1 unit.

   Therefore, the final answer is,

   There is a stretch in the x-direction by a stretch factor of 2. Followed by a translation in the y-direction by 1 unit.

2. The graph of $y = f(x)$ is transformed to the graph $y = 2f(x - 1)$. Describe fully the two single transformations which have been combined to give the resulting transformation. (9709/12/M/J/21 number 2)

   Start by describing the transformation in the x-axis,

   There is a translation in the x-direction by 1 unit.

   Then describe the transformation in the y-axis,

   There is a stretch in the y-direction by a stretch factor of 2.

   Therefore, the final answer is,

   There is a translation in the x-direction by 1 unit. Followed by a stretch in the y-direction by a stretch factor of 2

3. Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

   $$f(x) = x^2 - 2x + 5$$
   $$g(x) = x^2 + 4x + 13$$

   (9709/13/M/J/21 number 6)

   (a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants.

Start by completing the square for both $f(x)$ and $g(x)$,

$$f(x) = x^2 - 2x + 5$$
$$f(x) = (x-1)^2 + 4$$

$$g(x) = x^2 + 4x + 13$$
$$g(x) = (x+2)^2 + 9$$

Let's proceed to the next part,

$$g(x) = f(x+p) + q$$

Let's evaluate the equation above. p represents a translation in the x-direction from $f(x)$ to $g(x)$. To translate from $f(x)$ to $g(x)$ i.e from -1 to 2, in the x-direction we have to move by 3 units. Therefore,

$$p = 3$$

q represents a translation in the y-direction from $f(x)$ to $g(x)$. To translate from $f(x)$ to $g(x)$ i.e from 4 to 9, in the y-direction we have to move by 5 units. Therefore,

$$q = 5$$

Therefore, the final answer is,

$$g(x) = f(x+3) + 5$$

(b) Describe fully the transformations which transform the graph of $y = f(x)$ to the graph of $y = g(x)$.

$$g(x) = f(x+3) + 5$$

Start by describing the transformation in the x-axis,

There is a translation in the x-direction by -3 units.

Then describe the transformation in the y-axis,

There is a translation in the y-direction by 5 units.

Therefore, the final answer is,

There is a translation in the x-direction by -3 units. Followed by a translation in the y-direction by 5 units.