GCE A Level Maths 9709

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1.1 Quadratics

In this topic, we will learn how to:

- solving by substitution, a pair of simultaneous equations, of which one is linear and one is quadratic

Solving Simultaneous Equations

To solve a simultaneous equation make one of the unknowns in the linear equation subject of the formula. Substitute this into the quadratic, then solve for the other unknown. Let's take a look at the following examples.

1. Solve simultaneously by substitution the following equations,

$$x + y + 1 = 0, \qquad x^2 + y^2 = 25$$

The first step is to make x the subject of the formula in the linear equation,

$$x + y + 1 = 0$$

$$x = -y - 1$$

Substitute $x = -y - 1$ into the second equation,

$$x^2 + y^2 = 25$$

$$(-y - 1)^2 + y^2 = 25$$

Expand the parentheses,

$$y^2 + 2y + 1 + y^2 = 25$$

Group like terms,

$$y^2 + y^2 + 2y + 1 - 25 = 0$$

Simplify,

$$2y^2 + 2y - 24 = 0$$

Divide the whole equation by 2 to further simplify it,

$$y^2 + y - 12 = 0$$

Solve the quadratic equation using your preferred method, in this case, we will factorise,

$$(y + 4)(y - 3) = 0$$

$$y + 4 = 0 \qquad\qquad y - 3 = 0$$

Solve for y in both equations,

$$y = -4 \qquad\qquad y = 3$$

Substitute these values of y into the original linear equation to find the values of x. At $y = -4$,

$$x + {\color{blue}y} + 1 = 0$$

$$x + ({\color{blue}-4}) + 1 = 0$$

$$x - 4 + 1 = 0$$

$$x - 3 = 0$$

$$x = 3$$

Therefore, at $y = -4$, $x = 3$. We can represent this as a set of coordinates,

$$(3, -4)$$

We do the same for $y = 3$. At $y = 3$,

$$x + {\color{green}y} + 1 = 0$$

$$x + ({\color{green}3}) + 1 = 0$$

$$x + 3 + 1 = 0$$

$$x + 4 = 0$$

$$x = -4$$

Therefore, at $y = 3$, $x = -4$. We can represent this as a set of coordinates,

$$(-4, 3)$$

As a result, the final solution for this question is,

$$(3, -4), \qquad (-4, 3)$$

2. Solve simultaneously by substitution the following equations,

$$2x + 3y = 7 \qquad\qquad 3x^2 = 4 + 4xy$$

The first step is to make x the subject of the formula in the linear equation,

$$2x + 3y = 7$$

$$2x = 7 - 3y$$

$$x = \frac{7}{2} - \frac{3y}{2}$$

Substitute $x = \frac{7}{2} - \frac{3y}{2}$ into the second equation,

$$3x^2 = 4 + 4xy$$

$$3\left(\frac{7}{2} - \frac{3y}{2}\right)^2 = 4 + 4\left(\frac{7}{2} - \frac{3y}{2}\right)y$$

Expand the parentheses,

$$3\left(\frac{49}{4} - \frac{21y}{2} + \frac{9y^2}{4}\right) = 4 + 14y - 6y^2$$

$$\frac{147}{4} - \frac{63y}{2} + \frac{27y^2}{4} = 4 + 14y - 6y^2$$

Group like terms,

$$\frac{147}{4} - 4 - \frac{63y}{2} - 14y + \frac{27y^2}{4} + 6y^2 = 0$$

Simplify,

$$\frac{131}{4} - \frac{91y}{2} + \frac{51y^2}{4} = 0$$

Multiply through by 4 to get to rid of the denominators,

$$131 - 182y + 51y^2 = 0$$

Write the equation in the form $ax^2 + bx + c = 0$,

$$51y^2 - 182y + 131 = 0$$

Solve the quadratic equation using your preferred method, in this case we will factorise,

$$(51y - 131)(y - 1) = 0$$

$$51y - 131 = 0 \qquad\qquad y - 1 = 0$$

Solve for y in both equations,

$$y = \frac{131}{51} \qquad\qquad y = 1$$

Substitute these values of y into the original linear equation to find the values of x. At $y = \frac{131}{51}$,

$$2x + 3y = 7$$

$$2x + 3\left(\frac{131}{51}\right) = 7$$

$$2x + \frac{131}{17} = 7$$

$$2x = 7 - \frac{131}{17}$$

$$2x = -\frac{12}{17}$$

$$x = -\frac{6}{17}$$

Therefore, at $y = \frac{131}{51}$, $x = -\frac{6}{17}$. We can represent this as a set of coordinates,

$$\left(-\frac{6}{17}, \frac{131}{51}\right)$$

We do the same for $y = 1$. At $y = 1$,

$$2x + 3y = 7$$

$$2x + 3\left(1\right) = 7$$

$$2x + 3 = 7$$

$$2x = 7 - 3$$

$$2x = 4$$

$$x = 2$$

Therefore, at $y = 1$, $x = 2$. We can represent this as a set of coordinates,

$$(2, 1)$$

As a result, the final solution for this question is,

$$\left(-\frac{6}{17}, \frac{131}{51}\right), \qquad (2, 1)$$