Pure Maths 1

1.1 Quadratics - Easy

SMTYL

GCEALevelMaths9709

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Pure Mathematics 1**
Topic:	**1.1 Quadratics**
Difficulty:	**Easy**

Questions

1. (a) Express $4x^2 - 24x + p$ in the form $a(x+b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. (9709/12/M/J/23 number 3)
   (b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots.
2. Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. (9709/12/F/M/22 number 5a)
3. (a) Express $x^2 - 8x + 11$ in the form $(x+p)^2 + q$ where p and q are constants. (9709/11/M/J/2022 number 1)
   (b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$.
4. The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$. Express $f(x)$ in the form $2(x+a)^2 + b$. (9709/13/M/J/22 number 6a)
5. Solve the equation $3x + 2 = \frac{2}{x-1}$. (9709/11/O/N/22 number 1)
6. The functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9$$

$$g(x) = 2x^2 + 4x + 12$$

   (9709/11/O/N/22 number 9ab)
   (a) Express $f(x)$ in the form $(x-a)^2 + b$.
   (b) Express $g(x)$ in the form $2\left[(x+c)^2 + d\right]$.
7. Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. (9709/12/O/N/22 number 3a)
8. The equation of a curve is $y = 4x^2 + 20x + 6$. (9709/12/O/N/22 number 6)
   (a) Express the equation in the form $a(x+b)^2 + c$, where a, b and c are constants.
   (b) Hence solve the equation $4x^2 + 20x + 6 = 45$.
   (c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y- axes.
9. The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$. Express $f(x)$ in the form $-2(x+a)^2 + b$, where a and b are integers. (9709/13/O/N/22 number 2a)
10. (a) Express $16x^2 - 24x + 10$ in the form $(4x+a)^2 + b$. (9709/12/M/J/21 number 1)
    (b) It is given that the equation $16x^2 - 24x + 10 = k$ where k is a constant, has exactly one root. Find the value of this root.
11. Express $-3x^2 + 12x + 2$ in the form $-3(x-a)^2 + b$, where a and b are constants. (9709/11/O/N/21 number 8a)
12. Express $5y^2 - 30y + 50$ in the form $5(y+a)^2 + b$, where a and b are constants. (9709/13/O/N/21 number 3a)
13. Express $2x^2 + 12x + 11$ in the form $2(x+a)^2 + b$, where a and b are constants. (9709/12/F/M/20 number 9a)
14. Express $x^2 + 6x + 5$ in the form $(x+a)^2 + b$, where a and b are constants. (9709/13/O/N/20 number 1a)

Answers

1. (a) Express $4x^2 - 24x + p$ in the form $a(x+b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. (9709/12/M/J/23 number 3)

$$4x^2 - 24x + p$$

Start by factoring out the 4 from the first two terms,

$$4\left(x^2 - 6x\right) + p$$

Complete the square for the quadratic inside the parentheses,

$$4\left[(x-3)^2 - 9\right] + p$$

Expand the square brackets,

$$4(x-3)^2 - 36 + p$$

Therefore, the final answer is,

$$4(x-3)^2 - 36 + p$$

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots.

$$4x^2 - 24x + p = 0$$

Since the equation has no real roots, we can say that,

$$b^2 - 4ac < 0$$

Identify the values of a, b and c from the given equation,

$$a = 4, b = -24, c = p$$

Substitute these values into the discriminant,

$$(-24)^2 - 4(4)(p) < 0$$

Solve the inequality,

$$576 - 16p < 0$$

$$16p > 576$$

$$p > 36$$

Therefore, the final answer is,

$$p > 36$$

2. Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. (9709/12/F/M/22 number 5a)

$$2x^2 - 8x + 14$$

Start by factoring out the 2 from all the terms,

$$2\left(x^2 - 4x + 7\right)$$

Complete the square for the quadratic inside the parentheses,

$$2\left[(x-2)^2 - 4 + 7\right]$$

Simplify inside the square brackets,

$$2\left[(x-2)^2 + 3\right]$$

Therefore, the final answer is,

$$2\left[(x-2)^2 + 3\right]$$

3. (a) Express $x^2 - 8x + 11$ in the form $(x+p)^2 + q$ where p and q are constants. (9709/11/M/J/2022 number 1)

$$x^2 - 8x + 11$$

For the quadratic $x^2 + bx + c$, the completed square form is,

$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right) + c$$

Identify b and c,

$$b = -8, c = 11$$

Substitute into the completed square form,

$$\left(x + \frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right) + 11$$

Simplify,

$$(x-4)^2 - 16 + 11$$
$$(x-4)^2 - 5$$

Therefore, the final answer is,

$$(x-4)^2 - 5$$

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$.

$$x^2 - 8x + 11 = 1$$

Replace the quadratic with its completed square form,

$$(x-4)^2 - 5 = 1$$

Now let's make x the subject of the formula. Start by moving -5 to the right hand side,

$$(x-4)^2 = 1+5$$

Simplify the right hand side,

$$(x-4)^2 = 6$$

Take the square root of both sides,

$$\sqrt{(x-4)^2} = \pm\sqrt{6}$$

Simplify,

$$x-4 = \pm\sqrt{6}$$

Move -4 to the right hand side,

$$x = 4 \pm \sqrt{6}$$

We have made x the subject of the formula. Therefore, the final answer is,

$$x = 4 \pm \sqrt{6}$$

4. The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$. Express $f(x)$ in the form $2(x+a)^2 + b$. (9709/13/M/J/22 number 6a)

$$2x^2 - 16x + 23$$

Start by factoring out the 2 from the first two terms,

$$2\left(x^2 - 8x\right) + 23$$

Complete the square for the quadratic inside the parentheses,

$$2\left[(x-4)^2 - 16\right] + 23$$

Expand the square brackets,

$$2(x-4)^2 - 32 + 23$$

Simplify,

$$2(x-4)^2 - 9$$

Therefore, the final answer is,

$$2(x-4)^2 - 9$$

5. Solve the equation $3x + 2 = \frac{2}{x-1}$. (9709/11/O/N/22 number 1)

$$3x + 2 = \frac{2}{x-1}$$

Get rid of the denominator by multiplying both sides by $x - 1$,

$$(3x + 2)(x - 1) = 2$$

Expand that the brackets on the left hand side,

$$3x^2 - 3x + 2x - 2 = 2$$

Simplify the left hand side,

$$3x^2 - x - 2 = 2$$

Put all terms on one side,

$$3x^2 - x - 2 - 2 = 0$$

Simplify the left hand side,

$$3x^2 - x - 4 = 0$$

Solve the quadratic using your method of choice. In this example we will factorize,

$$(3x - 4)(x + 1) = 0$$

Equate each bracket to 0,

$$3x - 4 = 0 \qquad x + 1 = 0$$

Solve the two linear equations,

$$x = \frac{4}{3} \qquad x = -1$$

Therefore, the final answer is,

$$x = -1, \frac{4}{3}$$

6. The functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9$$
$$g(x) = 2x^2 + 4x + 12$$

(9709/11/O/N/22 number 9ab)

(a) Express $f(x)$ in the form $(x-a)^2 + b$.

$$x^2 - 4x + 9$$

For the quadratic $x^2 + bx + c$, the completed square form is,

$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right) + c$$

Identify b and c,

$$b = -4, c = 9$$

Substitute into the completed square form,

$$\left(x + \frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right) + 9$$

Simplify,

$$(x-2)^2 - 4 + 9$$
$$(x-2)^2 + 5$$

Therefore, the final answer is,

$$(x-2)^2 + 5$$

(b) Express $g(x)$ in the form $2\left[(x+c)^2 + d\right]$.

$$2x^2 + 4x + 12$$

Start by factoring out the 2 from all the terms,

$$2(x^2 + 2x + 6)$$

Complete the square for the quadratic inside the parentheses,

$$2\left[(x+1)^2 - 1 + 6\right]$$

Simplify inside the square brackets,

$$2\left[(x+1)^2 + 5\right]$$

Therefore, the final answer is,

$$2\left[(x+1)^2 + 5\right]$$

7. Find the set of values of k for which the equation $8x^2+kx+2 = 0$ has no real roots. (9709/12/O/N/22 number 3a)

$$8x^2 + kx + 2 = 0$$

Since we are told that the equation has no real roots, we know that,

$$b^2 - 4ac < 0$$

Identify the values of a, b and c,

$$a = 8, b = k, c = 2$$

Substitute these values into the discriminant,

$$k^2 - 4(8)(2) < 0$$

Simplify the left hand side,

$$k^2 - 64 < 0$$

Find the roots of the quadratic,

$$(k-8)(k+8) = 0$$

$$k = \pm 8$$

Identify the desired region for the quadratic inequality,

$$-8 < k < 8$$

Therefore, the final answer is,

$$-8 < k < 8$$

8. The equation of a curve is $y = 4x^2 + 20x + 6$. (9709/12/O/N/22 number 6)

   (a) Express the equation in the form $a(x+b)^2 + c$, where a, b and c are constants.

$$4x^2 + 20x + 6$$

Start by factoring out the 4 from the first two terms,

$$4(x^2 + 5x) + 6$$

Complete the square for the quadratic inside the parentheses,

$$4\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 6$$

Expand the square brackets,

$$4\left(x + \frac{5}{2}\right)^2 - 25 + 6$$

Simplify,

$$4\left(x+\frac{5}{2}\right)^2-19$$

Therefore, the final answer is,

$$4\left(x+\frac{5}{2}\right)^2-19$$

(b) Hence solve the equation $4x^2+20x+6=45$.

$$4x^2+20x+6=45$$

Replace the quadratic with the completed square form,

$$4\left(x+\frac{5}{2}\right)^2-19=45$$

Now we have to make x the subject of the formula. Start by moving -19 to the right hand side,

$$4\left(x+\frac{5}{2}\right)^2=45+19$$

Simplify the right hand side,

$$4\left(x+\frac{5}{2}\right)^2=64$$

Divide both sides by 4,

$$\left(x+\frac{5}{2}\right)^2=16$$

Take the square root of both sides,

$$\sqrt{\left(x+\frac{5}{2}\right)^2}=\pm\sqrt{16}$$

Simplify both sides,

$$x+\frac{5}{2}=\pm4$$

Move $\frac{5}{2}$ to the right hand side,

$$x=-\frac{5}{2}\pm4$$

Simplify,

$$x=\frac{-13}{2},\frac{3}{2}$$

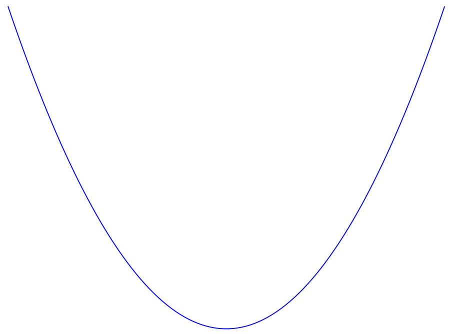
Therefore, the final answer is,

$$x = \frac{-13}{2}, \frac{3}{2}$$

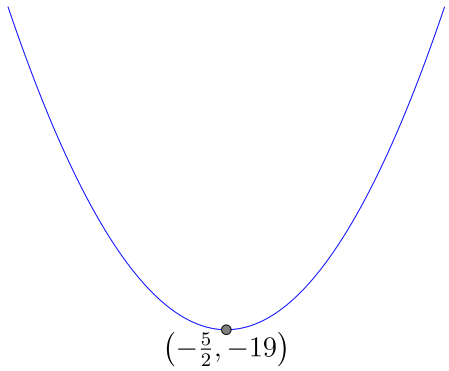
(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y- axes.

$$4\left(x + \frac{5}{2}\right)^2 - 19$$

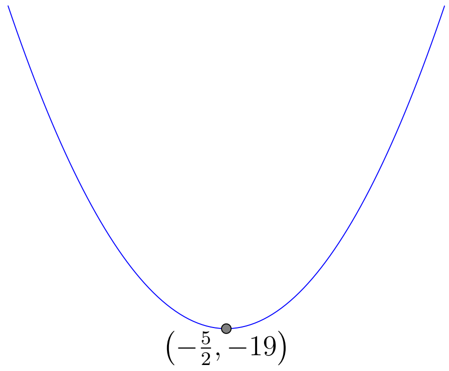
The coefficient of x^2 is positive so we know that we have a u-shaped parabola,



Label the stationary point,



We are not required to show the axes. Therefore, the final answer is,



9. The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$. Express $f(x)$ in the form $-2(x+a)^2 + b$, where a and b are integers. (9709/13/O/N/22 number 2a)

$$-2x^2 - 8x - 13$$

Start by factoring out the -2 from the first two terms,

$$-2\left(x^2 + 4x\right) - 13$$

Complete the square for the quadratic inside the parentheses,

$$-2\left[(x+2)^2 - 4\right] - 13$$

Expand the square brackets,

$$-2(x+2)^2 + 8 - 13$$

Simplify,

$$-2(x+2)^2 - 5$$

Therefore, the final answer is,

$$-2(x+2)^2 - 5$$

10. (a) Express $16x^2 - 24x + 10$ in the form $(4x+a)^2 + b$. (9709/12/M/J/21 number 1)

$$16x^2 - 24x + 10$$

Start by factoring out the 16 from the first two terms,

$$16\left(x^2 - \frac{3}{2}x\right) + 10$$

Complete the square for the quadratic inside the parentheses,

$$16\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 10$$

Expand the square brackets,

$$16\left(x - \frac{3}{4}\right)^2 - 9 + 10$$

Simplify,

$$16\left(x - \frac{3}{4}\right)^2 + 1$$

Rewrite 16 in terms of its square root,

$$4^2\left(x - \frac{3}{4}\right)^2 + 1$$

Now you can multiply 4 into the bracket since it is raised to the same power as the bracket,

$$\left[4\left(x-\frac{3}{4}\right)\right]^2+1$$

Expand inside the square brackets,

$$[4x-3]^2+1$$

Therefore, the final answer is,

$$(4x-3)^2+1$$

(b) It is given that the equation $16x^2-24x+10=k$ where k is a constant, has exactly one root. Find the value of this root.

$$16x^2-24x+10=k$$

Put all the terms on one side,

$$16x^2-24x+10-k=0$$

Since the equation has one root, we know that,

$$b^2-4ac=0$$

Identify the values of a, b and c,

$$a=16,-24,10-k$$

Substitute these values into the discriminant,

$$(-24)^2-4(16)(10-k)=0$$

Solve the linear equation,

$$576-64(10-k)=0$$

Expand the bracket,

$$576-640+64k=0$$

Simplify the left hand side,

$$-64+64k=0$$

Make k the subject of the formula,

$$64k=64$$

$$k=1$$

Now that we have the value of k, let's use the completed square form to find the root,

$$(4x-3)^2+1=k$$

$$(4x-3)^2+1=1$$

Now let's make x the subject of the formula. Start by subtracting 1 from both sides,

$$(4x-3)^2 = 0$$

Take the square root of both sides,

$$4x-3=0$$

Make x the subject of the formula,

$$4x = 3$$
$$x = \frac{3}{4}$$

Therefore, the final answer is,

$$x = \frac{3}{4}$$

11. Express $-3x^2+12x+2$ in the form $-3(x-a)^2+b$, where a and b are constants. (9709/11/O/N/21 number 8a)

$$-3x^2+12x+2$$

Start by factoring out the -3 from the first two terms,

$$-3\left(x^2-4x\right)+2$$

Complete the square for the quadratic inside the parentheses,

$$-3\left[(x-2)^2-4\right]+2$$

Expand the square brackets,

$$-3\left(x-2\right)^2+12+2$$

Simplify,

$$-3\left(x-2\right)^2+14$$

Therefore, the final answer is,

$$-3\left(x-2\right)^2+14$$

12. Express $5y^2-30y+50$ in the form $5(y+a)^2+b$, where a and b are constants. (9709/13/O/N/21 number 3a)

$$5y^2-30y+50$$

Start by factoring out the 5 from the first two terms,

$$5\left(y^2-6y\right)+50$$

Complete the square for the quadratic inside the parentheses,

$$5\left[(y-3)^2-9\right]+50$$

Expand the square brackets,

$$5\left(y-3\right)^2-45+50$$

Simplify,

$$5\left(y-3\right)^2+5$$

Therefore, the final answer is,

$$5\left(y-3\right)^2+5$$

13. Express $2x^2+12x+11$ in the form $2(x+a)^2+b$, where a and b are constants. (9709/12/F/M/20 number 9a)

$$2x^2+12x+11$$

Start by factoring out the 2 from the first two terms,

$$2\left(x^2+6x\right)+11$$

Complete the square for the quadratic inside the parentheses,

$$2\left[\left(x+3\right)^2-9\right]+11$$

Expand the square brackets,

$$5\left(x+3\right)^2-18+11$$

Simplify,

$$5\left(x+3\right)^2-7$$

Therefore, the final answer is,

$$5\left(x+3\right)^2-7$$

14. Express x^2+6x+5 in the form $(x+a)^2+b$, where a and b are constants. (9709/13/O/N/20 number 1a)

$$x^2+6x+5$$

For the quadratic x^2+bx+c, the completed square form is,

$$\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)+c$$

Identify b and c,

$$b=6, c=5$$

Substitute into the completed square form,

$$\left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)+5$$

Simplify,

$$(x+3)^2 - 9 + 5$$
$$(x+3)^2 - 4$$

Therefore, the final answer is,

$$(x+3)^2 - 4$$