Pure Maths 1

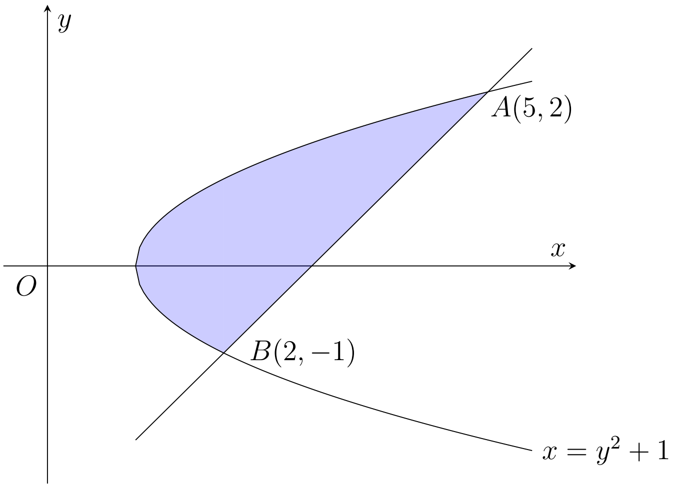
1.3 Coordinate Geometry - Easy

GCEALevelMaths9709

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Pure Mathematics 1**
Topic:	**1.3 Coordinate Geometry**
Difficulty:	**Easy**

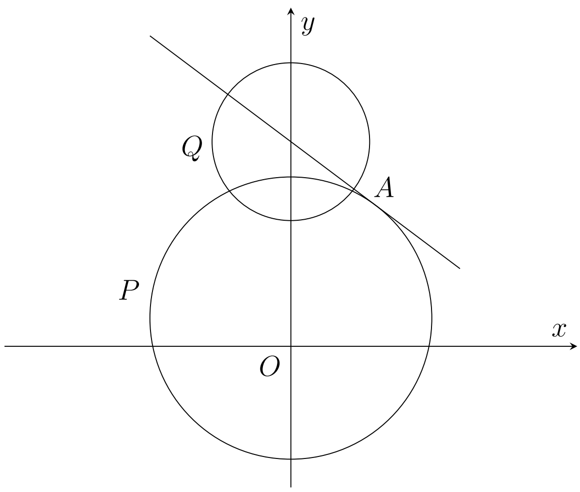
Questions

1.



The diagram shows the curve with equation $x = y^2 + 1$. The points $A(5, 2)$ and $B(2, -1)$ lie on the curve. Find an equation of the line AB. (9709/12/F/M/23 number 11a)

2.



The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$. (9709/11/M/J/23 number 12)

(a) Write down the equation of circle P.

(b) Find the equation of the tangent to the circle P at A.

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11.

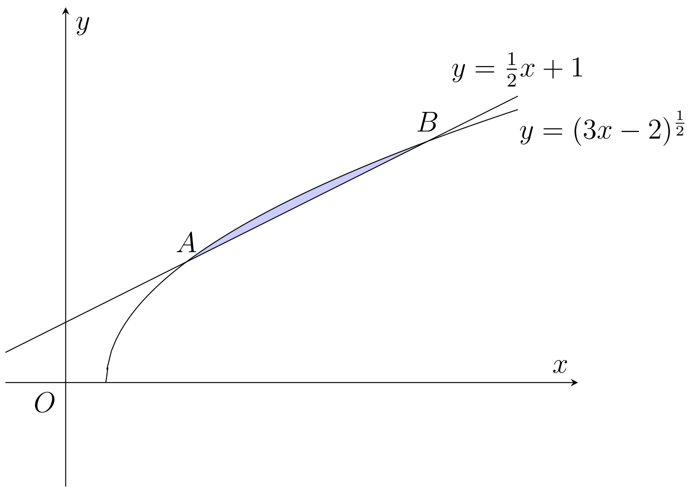
(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form.

3. A circle has equation $(x-1)^2+(y+4)^2=40$. A line with equation $y=x-9$ intersects the circle at points A and B. (9709/13/M/J/23 number 5)

   (a) Find the coordinates of the two points of intersection.

   (b) Find an equation of the circle with diameter AB.

4.



The diagram shows the curve with equation $y=(3x-2)^{\frac{1}{2}}$ and the line $y=\frac{1}{2}x+1$. The curve and the line intersect at points A and B. Find the coordinates of A and B. (9709/11/M/J/22 number 7a)

5. The equation of a circle is $x^2+y^2+6x-2y-26=0$. Find the coordinates of the centre of the circle and radius. Hence find the coordinates of the lowest point on the circle. (9709/11/M/J/22 number 9a)

6. Points A and B have coordinates $(5,2)$ and $(10,-1)$ respectively. (9709/12/O/N/22 number 1)

   (a) Find the equation of the perpendicular bisector of AB.

   (b) Find the equation of the circle with centre A that passes through B.

7. The equation of a curve is $y=(x-3)\sqrt{x+1}+3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places. (9709/12/M/J/21 number 3ab)

$$A(2,k)\quad B(2.9,2.8025)\quad C(2.99,2.9800)\quad D(2.999,2.9980)\quad E(3,3)$$

   (a) Find k, giving your answer correct to 4 decimal places.

   (b) Find the gradient of AE, giving your answer correct to 4 decimal places.

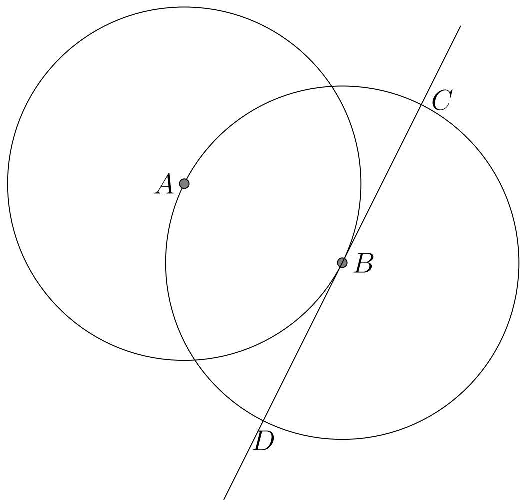
8. Points A and B have coordinates $(8,3)$ and (p,q) respectively. The equation of the perpendicular bisector of AB is $y=-2x+4$. Find the values of p and q. (9709/12/M/J/21 number 6)

9. The point A has coordinates $(1,5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has center $(5,11)$ and radius $\sqrt{52}$. (9709/12/M/J/21 number 7)

   (a) Show that l is a tangent to the circle at A.

   (b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also a tangent to A.

10.



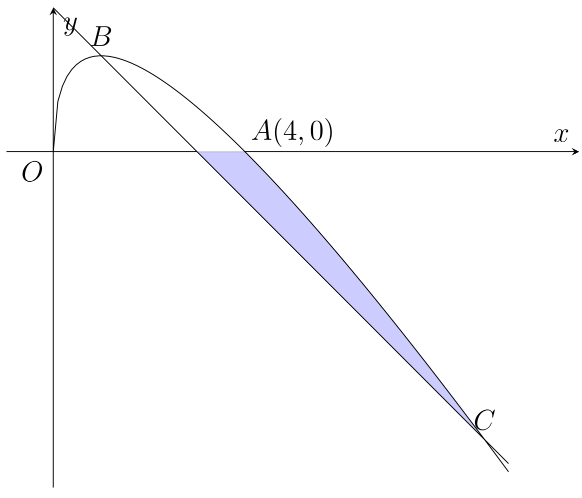
The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D. The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$. (9709/11/O/N/20 number 9)

(a) Find the equation of the tangent CBD.

(b) Find an equation of the circle with centre B.

(c) Find, by calculation the x-coordinates of C and D.

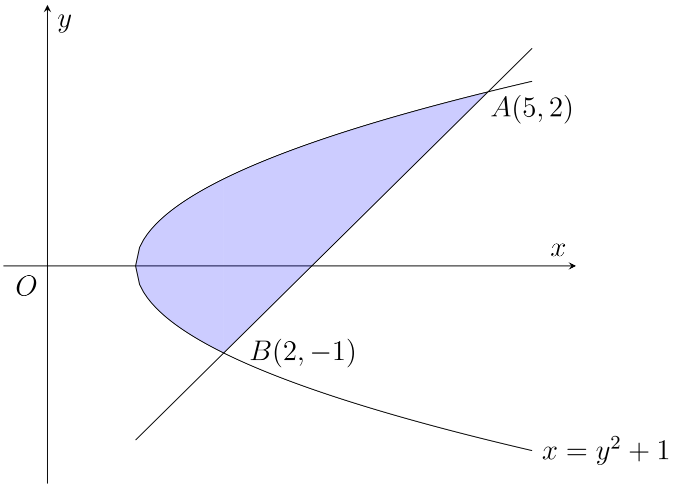
11.



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geq 0$ and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A(4, 0)$ and crosses the straight line at B and C. Find, by calculation, the x-coordinates of B and C. (9709/11/O/N/20 number 12a)

Answers

1.



The diagram shows the curve with equation $x = y^2 + 1$. The points $A(5,2)$ and $B(2,-1)$ lie on the curve. Find an equation of the line AB. (9709/12/F/M/23 number 11a)

$$A(5,2) \quad B(2,-1)$$

To find the equation of the line we need to find its gradient. Let's find the gradient of AB,

$$m_{AB} = \frac{-1-2}{2-5}$$
$$m_{AB} = 1$$

Now that we have the gradient, let's use it together with a point $A(5,2)$ that lies on the line to find the equation of line AB,

$$y = mx + c, \quad m = 1, \quad \text{passing through } A(5,2)$$

$$2 = 1(5) + c$$
$$2 = 5 + c$$
$$c = -3$$

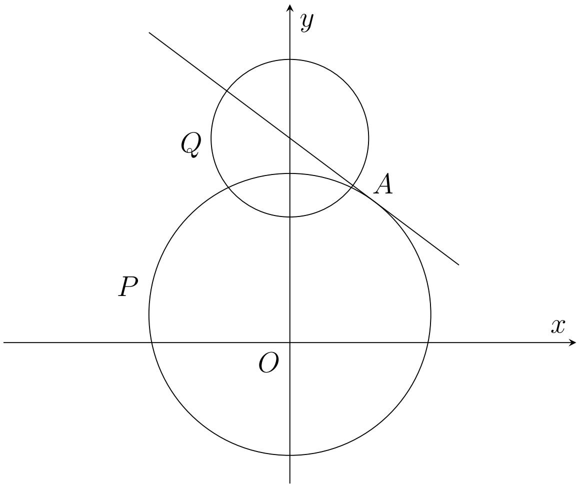
The equation of line AB is,

$$y = x - 3$$

Therefore, the final answer is,

$$y = x - 3$$

2.



The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$. (9709/11/M/J/23 number 12)

(a) Write down the equation of circle P.

centre of P $(0, 2)$ radius of P is 10

Let's write down the equation of the circle,

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x-0)^2 + (y-2)^2 = 10^2$$

$$x^2 + (y-2)^2 = 100$$

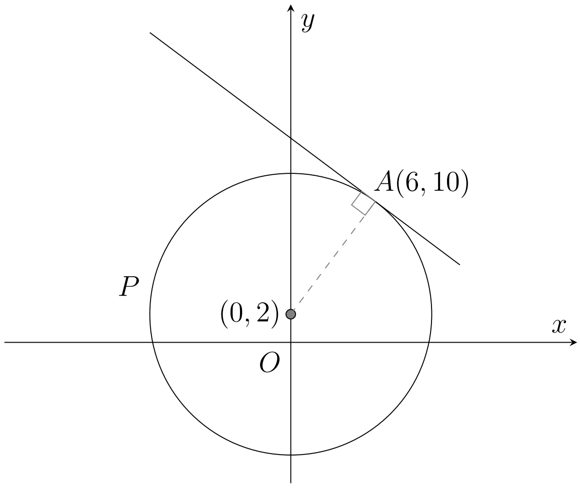
Therefore, the final answer is,

$$x^2 + (y-2)^2 = 100$$

(b) Find the equation of the tangent to the circle P at A.

$$x^2 + (y-2)^2 = 100 \quad A(6,10) \quad \text{centre of P}(0,2)$$

Let's sketch a diagram of the problem,



The radius from the centre to A is perpendicular to the tangent at A. Let's find the gradient of A by first finding the gradient of the radius,

$$m_{\text{radius}} = \frac{10-2}{6-0}$$

$$m_{\text{radius}} = \frac{4}{3}$$

Since the tangent is perpendicular to the radius, the gradient of the tangent must be the negative reciprocal,

$$m_{\text{tangent}} = -\frac{3}{4}$$

Now that we have the gradient of the tangent, we can find its equation,

$$y = mx + c, \quad m = -\frac{3}{4} \quad \text{passing through } A(6,10)$$

$$10 = -\frac{3}{4}(6) + c$$

$$10 = -\frac{9}{2} + c$$

$$c = 10 + \frac{9}{2}$$

$$c = \frac{29}{2}$$

The equation of the tangent is,

$$y = -\frac{3}{4}x + \frac{29}{2}$$

Therefore, the final answer is,

$$y = -\frac{3}{4}x + \frac{29}{2}$$

(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11.

$$y = -\frac{3}{4}x + \frac{29}{2}$$

In the stem of the question we are told that circle Q has its centre at the point where the tangent meets the y-axis. In other words its centre is the y-intercept,(c), of the tangent,

$$\left(0, \frac{29}{2}\right)$$

The radius is given as,

$$\frac{5}{2}\sqrt{5}$$

So the equation of the circle is,

$$(x-0)^2 + \left(y - \frac{29}{2}\right)^2 = \left(\frac{5}{2}\sqrt{5}\right)^2$$

$$x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$$

Now we have to verify that the y-coordinates of both the points of intersection of the two circles are 11,

$$x^2 + (y-2)^2 = 100 \quad x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$$

If we substitute $y = 11$ into both equations and they give two of the same values of x it means that $y = 11$ at both points of intersection,

$$x^2 + (11-2)^2 = 100 \quad x^2 + \left(11 - \frac{29}{2}\right)^2 = \frac{125}{4}$$

$$x^2 + 81 = 100 \quad x^2 + \frac{49}{4} = \frac{125}{4}$$

$$x^2 = 100 - 81 \quad x^2 = \frac{125}{4} - \frac{49}{4}$$

$$x^2 = 19 \quad x^2 = 19$$

$$x = \pm 19 \quad x = \pm 19$$

At $x = \pm 19$ the y-coordinates are both 11. This proves that the y-coordinates of both points of intersection are 11.

Therefore, the final answer is,

$$x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$$

At $y = 11$, $x = \pm 19$, hence the y-coordinates of both of the points of intersection of the two circles are 11.

(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form.

$$y = -\frac{3}{4}x + \frac{29}{2} \qquad x^2 + \left(y - \frac{29}{2}\right)^2 = \frac{125}{4}$$

To find the points of intersection we have to solve simultaneously. Substitute the linear equation into the equation of the circle,

$$x^2 + \left(-\frac{3}{4}x + \frac{29}{2} - \frac{29}{2}\right)^2 = \frac{125}{4}$$

Simplify the expression inside the brackets,

$$x^2 + \left(-\frac{3}{4}x\right)^2 = \frac{125}{4}$$

Expand the brackets,

$$x^2 + \frac{9}{16}x^2 = \frac{125}{4}$$

Simplify the equation and make x the subject of the formula,

$$\frac{25}{16}x^2 = \frac{125}{4}$$

$$\frac{16}{25} \times \frac{25}{16}x^2 = \frac{125}{4} \times \frac{16}{25}$$

$$x^2 = 20$$

$$x = \pm 2\sqrt{5}$$

Let's evaluate the y-coordinates,

$$y = -\frac{3}{4}x + \frac{29}{2}$$

At $x = -2\sqrt{5}$ At $x = 2\sqrt{5}$

$$y = -\frac{3}{4}\left(-2\sqrt{5}\right) + \frac{29}{2} \qquad y = -\frac{3}{4}\left(2\sqrt{5}\right) + \frac{29}{2}$$

$$y = \frac{3}{2}\sqrt{5} + \frac{29}{2} \qquad y = -\frac{3}{2}\sqrt{5} + \frac{29}{2}$$

Therefore, the final answer is,

$$\left(-2\sqrt{5}, \frac{3}{2}\sqrt{5}+\frac{29}{2}\right), \left(2\sqrt{5}, -\frac{3}{2}\sqrt{5}+\frac{29}{2}\right)$$

3. A circle has equation $(x-1)^2+(y+4)^2=40$. A line with equation $y=x-9$ intersects the circle at points A and B. (9709/13/M/J/23 number 5)

   (a) Find the coordinates of the two points of intersection.

$$(x-1)^2+(y+4)^2=40 \qquad y=x-9$$

To get the points of intersection we have to solve the two equations simultaneously. Substitute the linear equation into the equation of the circle,

$$(x-1)^2+(x-9+4)^2=40$$

Simplify the expression inside the brackets,

$$(x-1)^2+(x-5)^2=40$$

Expand the brackets,

$$x^2-2x+1+x^2-10x+25=40$$

Group like terms and simplify,

$$x^2+x^2-2x-10x+1+25-40=0$$

$$2x^2-12x-14=0$$

$$x^2-6x-7=0$$

Solve the quadratic equation,

$$(x-7)(x+1)=0$$

$$x=7 \quad x=-1$$

Evaluate the y-coordinates,

$$y=x-9$$

$$\text{At } x=7 \quad \text{At } x=-1$$

$$y=7-9 \quad y=-1-9$$

$$y=-2 \quad y=-10$$

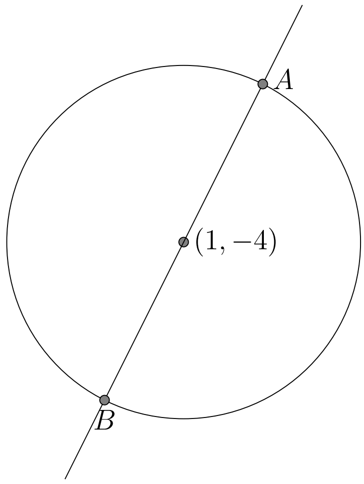
Therefore, the final answer is,

$$(7,-2),(-1,-10)$$

(b) Find an equation of the circle with diameter AB.

$$A(7,-2) \quad B(-1,-10)$$

Let's sketch a diagram of the problem,



Since AB is a diameter to the circle, the midpoint of AB is the centre of the circle. Let's find the midpoint of AB,

$$M_{AB} = \left(\frac{7-1}{2}, \frac{-2-10}{2}\right)$$

$$M_{AB} = (3,-6)$$

The coordinates of the centre of the circle are,

$$(3,-6)$$

Now we need to find the radius of the circle. Let's start by finding the length of diameter AB,

$$|AB| = \sqrt{(7+1)^2 + (-2+10)^2}$$

$$|AB| = 8\sqrt{2}$$

$$r = \frac{8\sqrt{2}}{4}$$

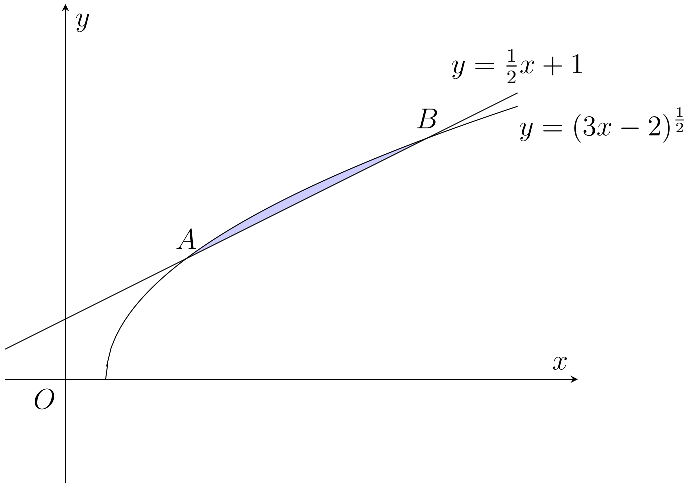
$$r = 4\sqrt{2}$$

$$r^2 = 32$$

Therefore, the final answer is,

$$(x-3)^2 + (y+6)^2 = 32$$

4.



The diagram shows the curve with equation $y = (3x-2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B. Find the coordinates of A and B. (9709/11/M/J/22 number 7a)

$$y = (3x-2)^{\frac{1}{2}} \qquad y = \frac{1}{2}x + 1$$

To get the points of intersection we have to solve the two equations simultaneously. Equate the two equations together,

$$(3x-2)^{\frac{1}{2}} = \frac{1}{2}x + 1$$

Square both sides to get rid of the power $\frac{1}{2}$,

$$3x - 2 = \left(\frac{1}{2}x + 1\right)^2$$

Expand the brackets,

$$3x - 2 = \frac{1}{4}x^2 + x + 1$$

Group like terms and simplify,

$$\frac{1}{4}x^2 + x - 3x + 1 + 2 = 0$$

$$\frac{1}{4}x^2 - 2x + 3 = 0$$

$$x^2 - 8x + 12 = 0$$

Solve the quadratic equation,

$$(x-2)(x-6) = 0$$

$$x = 2 \quad x = 6$$

Evaluate the y-coordinates,

$$y = \frac{1}{2}x + 1$$

$$\text{At } x = 2 \quad \text{At } x = 6$$

$$y = \frac{1}{2}(2) + 1 \quad y = \frac{1}{2}(6) + 1$$

$$y = 2 \quad y = 4$$

Therefore, the final answer is,

$$(2, 2), (6, 4)$$

5. The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$. Find the coordinates of the centre of the circle and radius. Hence find the coordinates of the lowest point on the circle. (9709/11/M/J/22 number 9a)

$$x^2 + y^2 + 6x - 2y - 26 = 0$$

Move the constants to the right hand side,

$$x^2 + y^2 + 6x - 2y = 26$$

Put all terms containing any power in x next to each other and likewise for y,

$$x^2 + 6x + y^2 - 2y = 26$$

Complete the square for the terms in x as well as for the terms in y,

$$(x + 3)^2 - 9 + (y - 1)^2 - 1 = 26$$

Move all constants to the right hand side,

$$(x + 3)^2 + (y - 1)^2 = 26 + 9 + 1$$

Simplify,

$$(x + 3)^2 + (y - 1)^2 = 36$$

We now have our equation in the form,

$$(x - a)^2 + (y - b)^2 = r^2$$

Deduce the coordinates of the centre from the equationf,

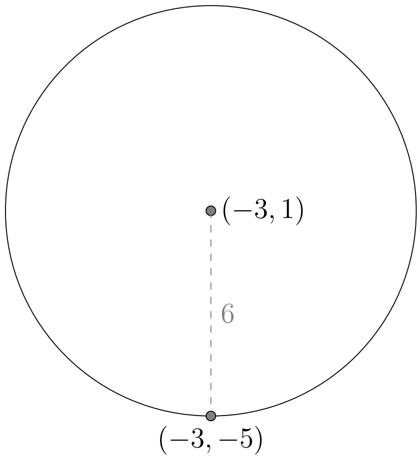
$$(-3, 1)$$

We can get the radius from r^2 in the equation of the circle,

$$r^2 = 36$$

$$r = 6$$

To find the lowest point on the circle we simply move straight down from the centre by the length of the radius,



Therefore, the final answer is,

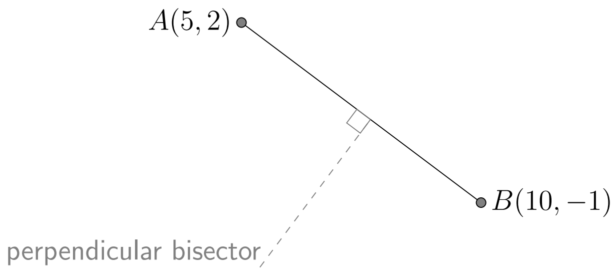
coordinates of the centre are $(-3, 1)$, $r = 6$, the coordinates of the lowest point on the circle are $(-3, -5)$

6. Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively. (9709/12/O/N/22 number 1)

   (a) Find the equation of the perpendicular bisector of AB.

$$A(5, 2) \quad B(10, -1)$$

Let's sketch a diagram of the problem,



The bisector is perpendicular to AB. Let's find the gradient of the bisector by first finding the gradient of AB,

$$m_{AB} = \frac{-1 - 2}{10 - 5}$$
$$m_{AB} = -\frac{3}{5}$$

Since the bisector is perpendicular to AB, the gradient of the bisector must be the negative reciprocal,

$$m_{\text{bisector}} = \frac{5}{3}$$

To find the equation of the bisector we also need a point that lies on the bisector. Since the bisector cuts AB into half, the midpoint of AB lies on the bisector. Let's find the midpoint of AB,

$$M_{AB} = \left(\frac{5+10}{2}, \frac{2-1}{2}\right)$$

$$M_{AB} = \left(\frac{15}{2}, \frac{1}{2}\right)$$

We have the gradient and a point that lies on the bisector, we can now find the equation of the perpendicular bisector,

$$y = mx + c, \quad m = \frac{5}{3}, \quad \text{passing through } M\left(\frac{15}{2}, \frac{1}{2}\right)$$

$$\frac{1}{2} = \frac{5}{3}\left(\frac{15}{2}\right) + c$$

$$\frac{1}{2} = \frac{25}{2} + c$$

$$c = -12$$

The equation of the perpendicular bisector is,

$$y = \frac{5}{3}x - 12$$

Therefore, the final answer is,

$$y = \frac{5}{3}x - 12$$

(b) Find the equation of the circle with centre A which passes through B.

$$A(5, 2) \quad B(10, -1)$$

The coordinates of the centre of the circle are,

$$(5, 2)$$

We need to find the radius of the circle. Since the circle has centre A and passes through B, the length of AB is the radius,

$$|AB| = \sqrt{(5-10)^2 + (2-(-1))^2}$$

$$|AB| = \sqrt{34}$$

$$r = \sqrt{34}$$

$$r^2 = 34$$

Now that we have the centre and r^2, we can write the equation of the circle,

$$(x-5)^2 + (y-2)^2 = 34$$

Therefore, the final answer is,

$$(x-5)^2+(y-2)^2=34$$

7. The equation of a curve is $y=(x-3)\sqrt{x+1}+3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places. (9709/12/M/J/21 number 3ab)

$$A(2,k)\quad B(2.9,2.8025)\quad C(2.99,2.9800)\quad D(2.999,2.9980)\quad E(3,3)$$

(a) Find k, giving your answer correct to 4 decimal places.

$$y=(x-3)\sqrt{x+1}+3\quad A(2,k)$$

Substitute the x in the equation with 2,

$$y=(2-3)\sqrt{2+1}+3$$

Evaluate y,

$$y=3-\sqrt{3}$$

Give your answer correct to 4 decimal places,

$$y=1.2679$$

Therefore, the final answer is,

$$k=1.2679$$

(b) Find the gradient of AE, giving your answer correct to 4 decimal places.

$$A(2,3-\sqrt{3})\qquad E(3,3)$$

Substitute the coordinates of A and E into the formula for gradient,

$$m_{AE}=\frac{3-(3-\sqrt{3})}{3-2}$$

Note: Use exact value of k for the calculation to get the most accurate answer.

$$m_{AE}=-\sqrt{3}$$

Give your answer correct to 4 decimal places,

$$m_{AE}=-1.7321$$

Therefore, the final answer is,

$$m_{AE}=-1.7321$$

8. Points A and B have coordinates $(8,3)$ and (p,q) respectively. The equation of the perpendicular bisector of AB is $y=-2x+4$. Find the values of p and q. (9709/12/M/J/21 number 6)

$$A(8,3),\quad B(p,q),\quad \text{eqn of perp bisec } AB\ y=-2x+4$$

To find p and q we need to create two equations in terms of p and q and solve them simultaneously. From the equation of the perpendicular bisector, we can tell that its gradient is,

$$m=-2$$

Since line AB is perpendicular to its perpendicular bisector the gradient of AB is the negative reciprocal,

$$m_{AB}=\frac{1}{2}$$

$$A(8,3),\ B(p,q)$$

Let's use the coordinates of A and B to create an equation in terms of p and q,

$$\frac{3-q}{8-p}=\frac{1}{2}$$

Simplify the equation,

$$2(3-q)=8-p$$

$$6-2q=8-p$$

$$p=8-6+2q$$

$$p=2+2q$$

The second equation will come from the midpoint of AB. We know that the bisector cuts AB in half, so the midpoint must lie on the perpendicular bisector. Substitute the coordinates of A and B into the formula for midpoint,

$$A(8,3),\ B(p,q)$$

$$M_{AB}=\left(\frac{8+p}{2},\frac{3+q}{2}\right)$$

$$M_{AB}=\left(\frac{8+p}{2},\frac{3+q}{2}\right)$$

Since the midpoint lies on the perpendicular bisector, we can substitute its coordinates into the equation of the perpendicular bisector,

$$y=-2x+4$$

$$\frac{3+q}{2}=-2\left(\frac{8+p}{2}\right)+4$$

Simplify the equation,

$$\frac{3+q}{2} = -8 - p + 4$$
$$3 + q = -16 - 2p + 8$$
$$3 + q = -8 - 2p$$
$$2p + q = -8 - 3$$
$$2p + q = -11$$

We now have two equations in terms of p and q,

$$p = 2 + 2q \quad 2p + q = -11$$

Solve the two equations simultaneously,

$$2(2 + 2q) + q = -11$$
$$4 + 4q + q = -11$$
$$4 + 5q = -11$$
$$5q = -15$$
$$q = -3$$

Evaluate p,

$$p = 2 + 2q$$
$$p = 2 + 2(-3)$$
$$p = -4$$

Therefore, the final answer is,

$$p = -4, \quad q = -3$$

9. The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has center $(5, 11)$ and radius $\sqrt{52}$. (9709/12/M/J/21 number 7)

   (a) Show that l is a tangent to the circle at A.

   $$\text{center } (5, 11) \quad A(1, 5) \quad m_l = -\frac{2}{3}$$

   If l is a tangent to the circle then the product of the gradient of the radius to A and the gradient of l should be -1,

   $$m_l \times m_{\text{radius}} = -1$$

   Let's find the gradient of the radius to A,

   $$m_{\text{radius}} = \frac{5 - 11}{1 - 5}$$
   $$m_{\text{radius}} = \frac{3}{2}$$

Let's check if the gradients satisfy the equation,

$$m_l \times m_{\text{radius}} = -1$$

$$-\frac{2}{3} \times \frac{3}{2} = -1$$

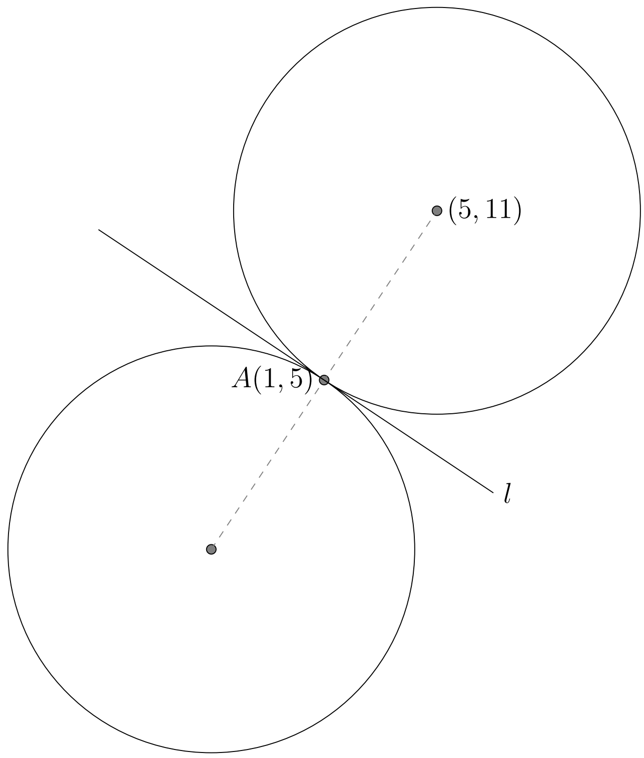
$m_l \times m_{\text{radius}}$ **is indeed equal to** -1**. Therefore, we have proved that,**

$$l \text{ is a tangent to the circle at } A$$

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also a tangent to A.

$$A(1,5) \quad r = \sqrt{52}$$

Let's sketch a diagram of the problem,



For this new circle to also have a tangent at A and the same radius it must be a reflection of the original circle. We already have the its radius we need to find the coordinates of the centre. Since the new circle is a reflection of the original circle, A is the midpoint of the centres of the two circles. Let's use the equation for midpoint to find the coordinates of centre,

$$A(1,5), \text{centre of original circle } (5,11), \text{centre of new circle } (x,y)$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(1,5) = \left(\frac{5 + x}{2}, \frac{11 + y}{2}\right)$$

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$$\frac{5+x}{2} = 1 \quad \frac{11+y}{2} = 5$$

$$5+x = 2 \quad 11+y = 10$$

$$x = -3 \quad y = -1$$

Therefore, the coordinates of the centre of the new circle are,

$$(-3, -1)$$

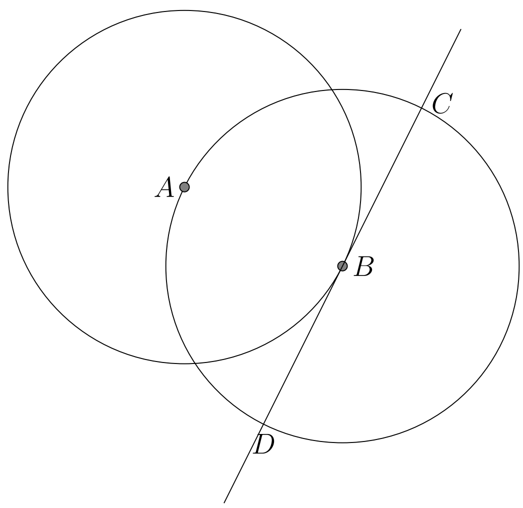
Now we can write the equation of the new circle,

$$(x+3)^2 + (y+1)^2 = 52$$

Therefore, the final answer is,

$$(x+3)^2 + (y+1)^2 = 52$$

10.



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D. The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$. (9709/11/O/N/20 number 9)

(a) Find the equation of the tangent CBD.

$$A(-1, 4) \quad B(3, 2)$$

Tangent CBD is perpendicular to AB. Let's find the gradient of tangent CBD by first finding the gradient of AB,

$$m_{AB} = \frac{4-2}{-1-3}$$
$$m_{AB} = -2$$

The gradient of tangent CBD is the negative reciprocal of the gradient of AB,

$$m_{CBD} = \frac{1}{2}$$

Now let's find the equation of the tangent CBD,

$$y = mx + c, \quad m = \frac{1}{2}, \text{ passes through } B(3,2)$$
$$2 = \frac{1}{2}(3) + c$$
$$2 = \frac{3}{2} + c$$
$$c = \frac{1}{2}$$

The equation of tangent CBD is,

$$y = \frac{1}{2}x + \frac{1}{2}$$

Therefore, the final answer is,

$$y = \frac{1}{2}x + \frac{1}{2}$$

(b) Find an equation of the circle with centre B.

$$A(-1,4) \quad B(3,2)$$

We already have the coordinates of the centre,

$$(3,2)$$

We need to find the radius. From the diagram, we can tell that the radius of the circle with centre B is equal to AB. Let's find the length of AB,

$$|AB| = \sqrt{(-1-3)^2 + (4-2)^2}$$
$$|AB| = 2\sqrt{5}$$
$$r = 2\sqrt{5}$$
$$r^2 = 20$$

The equation of the circle with centre B is,

$$(x-3)^2 + (y-2)^2 = 20$$

Therefore, the final answer is,

$$(x-3)^2+(y-2)^2=20$$

(c) Find, by calculation the x-coordinates of C and D.

$$y=\frac{1}{2}x+\frac{1}{2} \qquad (x-3)^2+(y-2)^2=20$$

C and D are the points of intersection of tangent CBD and the circle with centre B. Solve the two equations simultaneously to get the coordinates of C and D.

Substitute the linear equation into the equation of the circle,

$$(x-3)^2+\left(\frac{1}{2}x+\frac{1}{2}-2\right)^2=20$$

Simplify the expression inside the brackets,

$$(x-3)^2+\left(\frac{1}{2}x-\frac{3}{2}\right)^2=20$$

Expand the brackets,

$$x^2-6x+9+\frac{1}{4}x^2-\frac{3}{2}x+\frac{9}{4}=20$$

Group like terms and simplify,

$$x^2+\frac{1}{4}x^2-6x-\frac{3}{2}x+9+\frac{9}{4}-20=0$$

$$\frac{5}{4}x^2-\frac{15}{2}x-\frac{35}{4}=0$$

$$5x^2-30x-35=0$$

$$x^2-6x-7=0$$

Solve the quadratic equation,

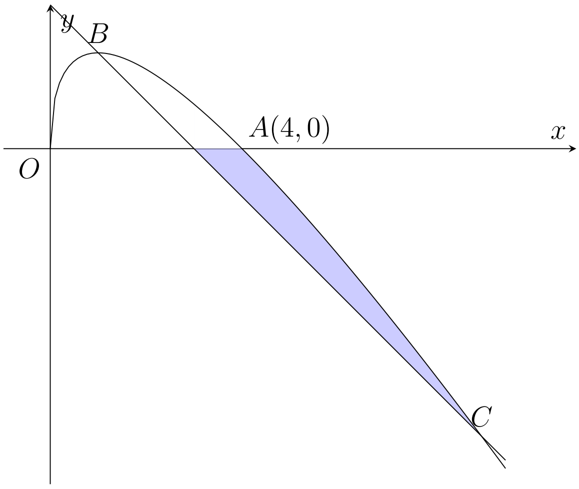
$$(x-7)(x+1)=0$$

$$x=7 \quad x=-1$$

Therefore, the final answer is,

$$x=7 \quad x=-1$$

11.



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geq 0$ and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A(4, 0)$ and crosses the straight line at B and C. Find, by calculation, the x-coordinates of B and C. (9709/11/O/N/20 number 12a)

$$y = 4x^{\frac{1}{2}} - 2x \qquad y = 3 - x$$

The line and the curve intersect at B and C. To find the x-coordinates of B and C we have to solve the equations simultaneously. Equate the two equations,

$$4x^{\frac{1}{2}} - 2x = 3 - x$$

Put all the terms on side and simplify,

$$2x - x - 4x^{\frac{1}{2}} + 3 = 0$$

$$x - 4x^{\frac{1}{2}} + 3 = 0$$

Notice that this is a hidden quadratic. Rewrite x in terms of $x^{\frac{1}{2}}$,

$$\left(x^{\frac{1}{2}}\right)^2 - 4x^{\frac{1}{2}} + 3 = 0$$

$$\left(x^{\frac{1}{2}}\right)^2 - 4x^{\frac{1}{2}} + 3 = 0$$

Solve the quadratic,

$$\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} - 3\right) = 0$$

$$x^{\frac{1}{2}} = 1 \quad x^{\frac{1}{2}}3$$

$$x = 1 \quad x = 9$$

Therefore, the final answer is,

$$x = 1, 9$$