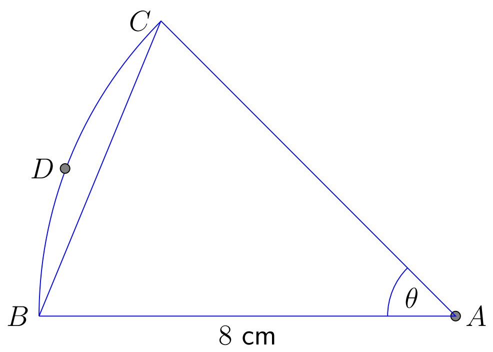
Pure Maths 1

1.4 Circular Measure - Easy

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Pure Mathematics 1**
Topic:	**1.4 Circular Measure**
Difficulty:	**Easy**

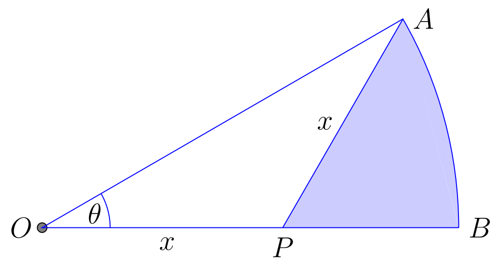
Questions

1.



The diagram shows a sector ABC of a circle with centre A and radius 8cm. The area of the sector is $\frac{16}{3}\pi$cm^2. The point D lies on the arc BC. Find the perimeter of the segment BCD. (9709/11/M/J/23 number 4)

2.

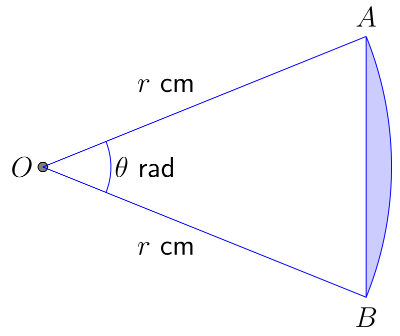


The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$. (9709/12/M/J/23 number 6)

(a) Show that the arc length AB is $2x\theta\cos\theta$.

(b) Find the area of the shaded region APB in terms of x and θ.

3.

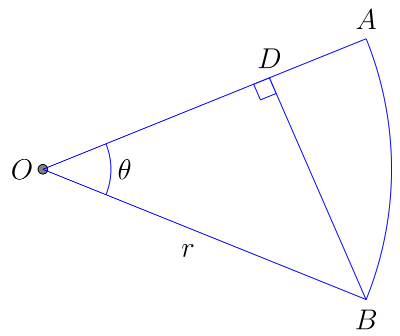


The diagram shows a sector OAB of a circle with centre O and radius $r$cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6cm and that the area of the sector OAB is 76.8cm^2. (9709/13/M/J/23 number 6)

(a) Find the area of the shaded region.

(b) Find the perimeter of the shaded region.

4.

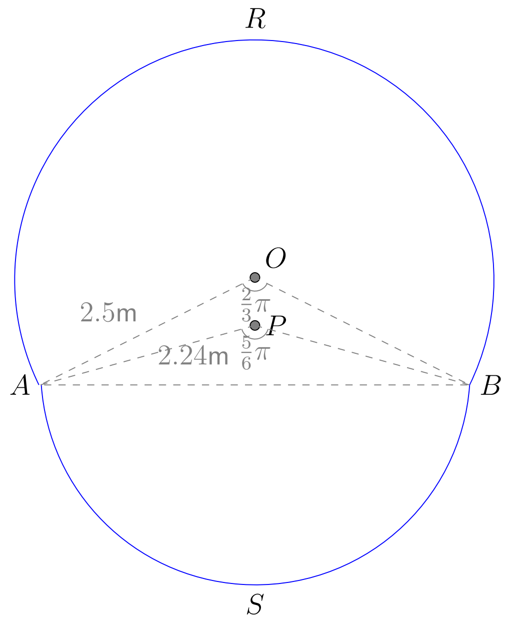


The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians. (9709/11/M/J/22 number 5)

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r.

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r.

5.



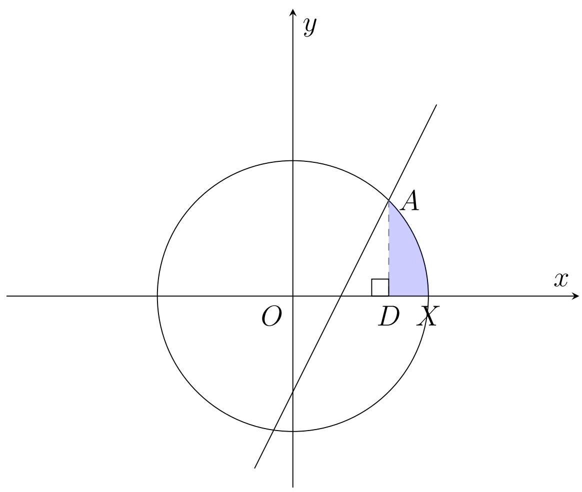
The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of a radius 2.5m, with centre O, a sector $PASB$ of another circle of radius 2.24m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$. (9709/12/O/N/22 number 10)

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places.

(b) Find the difference in area of the triangles AOB and APB, giving your answers correct to 2 decimal places.

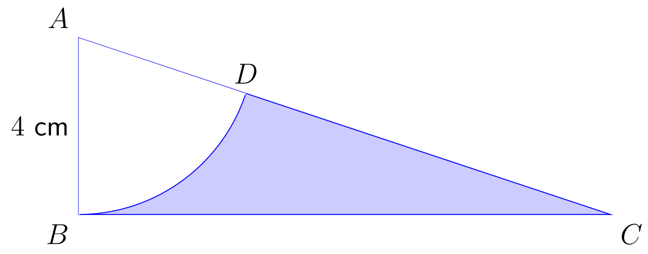
(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place.

6.



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis. Find an exact expression for the perimeter of the shaded region. (9709/13/O/N/22 number 10c)

7.

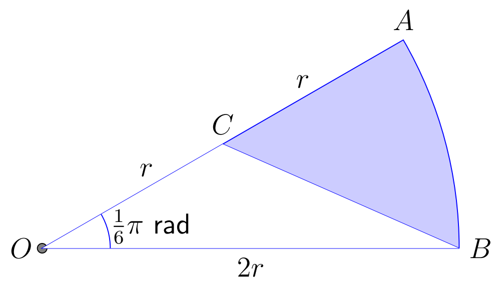


The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$cm. The sector ABD is part of a circle with centre A. The area of the sector is 10cm^2. (9709/13/M/J/21 number 5)

(a) Find angle BAD in radians.

(b) Find the perimeter of the shaded region.

8.



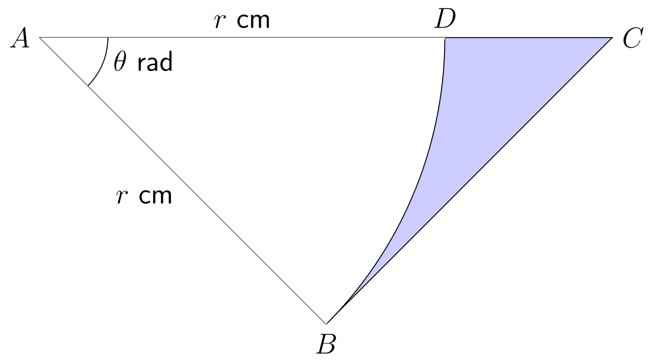
In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA. (9709/12/M/J/20 number 7)

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$.

(b) Find the exact perimeter of the shaded region.

(c) Find the exact area of the shaded region.

9.



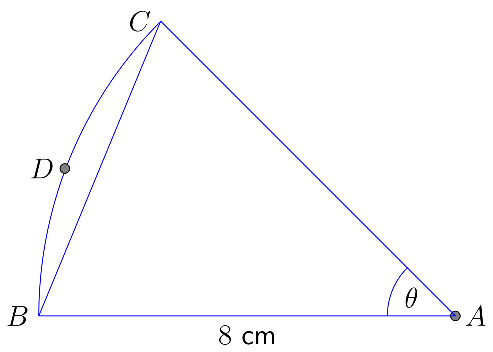
In the diagram, ABC is an isosceles triangle with $AB = BC = r$cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector a circle with centre A. (9709/12/O/N/20 number 8)

(a) Express the area of the shaded region in terms of r and θ.

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region.

Answers

1.



The diagram shows a sector ABC of a circle with centre A and radius 8cm. The area of the sector is $\frac{16}{3}\pi$cm^2. The point D lies on the arc BC. Find the perimeter of the segment BCD. (9709/11/M/J/23 number 4)

$$A = \frac{16}{3}\pi \quad r = 8$$

The perimeter we are looking for is,

$$\text{Perimeter} = BC + arc\ BC$$

Let's start by finding $arc\ BC$**,**

$$arc\ BC = r\theta$$

The radius we are given as 8**. To find theta let's use the information we are given about the area,**

$$A = \frac{16}{3}\pi$$

The formula for area is,

$$A = \frac{1}{2}r^2\theta$$

Substitute the values and solve for θ**,**

$$\frac{16}{3}\pi = \frac{1}{2} \times 8^2 \times \theta$$

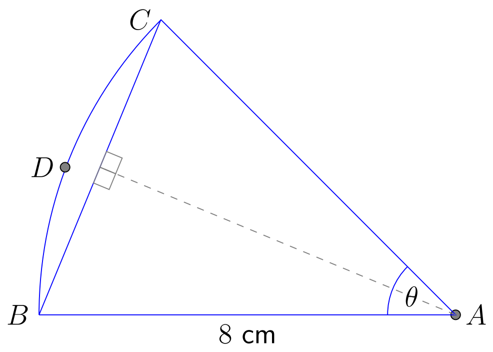
$$\frac{16}{3}\pi = 32\theta$$

$$\theta = \frac{1}{6}\pi$$

Now that we θ let's find $arc\ BC$,

$$arc\ BC = r\theta$$
$$arc\ BC = 8 \times \frac{1}{6}\pi$$
$$arc\ BC = \frac{4}{3}\pi$$

Now we need to find the length of line BC. Let's split the triangle into two right angled triangles,



We can use $SOHCAHTOA$ to find the length of BC,

$$\sin\left(\frac{1}{2} \times \frac{1}{6}\pi\right) = \frac{0.5BC}{8}$$
$$0.5BC = 8\sin\frac{1}{12}\pi$$
$$BC = 2 \times 8\sin\frac{1}{12}\pi$$
$$BC = 4\sqrt{6} - 4\sqrt{2}$$

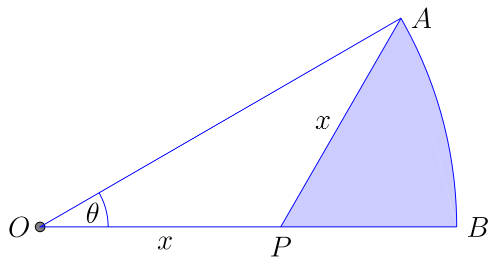
Now let's find the perimeter,

$$\text{Perimeter } = BC + arc\ BC$$
$$\text{Perimeter } = 4\sqrt{6} - 4\sqrt{2} + \frac{4}{3}\pi$$
$$\text{Perimeter } = 8.33$$

Therefore, the final answer is,

$$\text{Perimeter } = 8.33$$

2.



The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$. (9709/12/M/J/23 number 6)

(a) Show that the arc length AB is $2x\theta\cos\theta$.

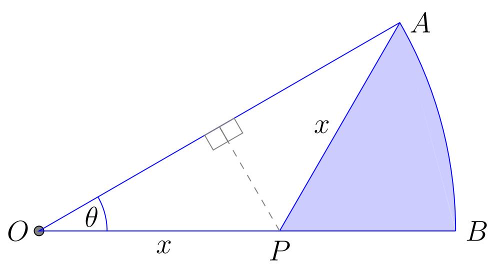
$$AOB = \theta \quad OP = AP = x$$

We are looking for $arc\ AB$**,**

$$arc\ AB = r\theta$$

We need to find the r **in terms of** θ **and** x**.**

Let's divide triangle OPA **into two right angled triangles,**



AO is equal to the radius. Let's use $SOHCAHTOA$ to find the length of AO,

$$\cos\theta = \frac{0.5AO}{x}$$

$$0.5AO = x\cos\theta$$

$$AO = 2x\cos\theta$$

$$r = 2x\cos\theta$$

Let's get back to the formula for arc length,

$$arc\ AB = r\theta$$

$$arc\ AB = 2x\cos(\theta)\times\theta$$

$$arc\ AB = 2x\theta\cos\theta$$

Therefore, the final answer is,

$$arc\ AB = 2x\theta\cos\theta$$

(b) Find the area of the shaded region APB in terms of x and θ.

$$r = 2x\cos\theta$$

Let's construct an equation to find the area of APB,

$$\text{Area of } APB = \text{Area of sector } AOB - \text{Area of triangle } OAP$$

Let's find the area of sector AOB,

$$A = \frac{1}{2}r^2\theta$$

$$\text{Area of sector } AOB = \frac{1}{2}\times(2x\cos\theta)^2\times\theta$$

$$\text{Area of sector } AOB = \frac{1}{2}\times 4x^2\cos^2\theta\times\theta$$

$$\text{Area of sector } AOB = 2x^2\theta\cos^2\theta$$

Let's find the area of triangle OAP,

$$A = \frac{1}{2}ab\sin C$$

$$\text{Area of triangle } OAP = \frac{1}{2}\times x\times x\times\sin(\pi-2\theta)$$

$$\text{Area of triangle } OAP = \frac{1}{2}x^2\sin(\pi-2\theta)$$

Now let's find the area of APB,

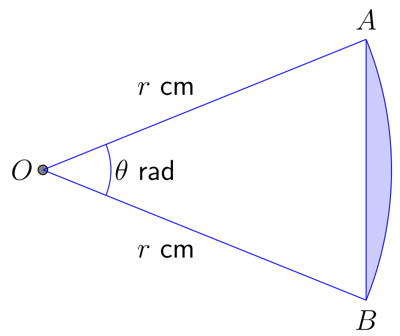
$$\text{Area of } APB = \text{Area of sector } AOB - \text{Area of triangle } OAP$$

$$\text{Area of } APB = 2x^2\theta\cos^2\theta - \frac{1}{2}x^2\sin(\pi-2\theta)$$

Therefore, the final answer is,

$$\text{Area of } APB = 2x^2\theta\cos^2\theta - \frac{1}{2}x^2\sin(\pi - 2\theta)$$

3.



The diagram shows a sector OAB of a circle with centre O and radius $r$cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6cm and that the area of the sector OAB is 76.8cm^2. (9709/13/M/J/23 number 6)

(a) Find the area of the shaded region.

$$arc\ AB = 9.6 \quad \text{Area of sector } OAB = 76.8$$

Before we find the area of the shaded region we need to find the values of r and θ. We can create two equations in terms of r and θ and solve them simultaneously,

$$arc\ AB = 9.6 \quad \text{Area of sector } OAB = 76.8$$
$$r\theta = 9.6 \quad \frac{1}{2}r^2\theta = 76.8$$

Make r the subject of the formula in the first equation,

$$r = \frac{9.6}{\theta}$$

Substitute into the second equation,

$$\frac{1}{2}r^2\theta = 76.8$$
$$\frac{1}{2}\left(\frac{9.6}{\theta}\right)^2\theta = 76.8$$
$$\frac{1}{2} \times \frac{92.16}{\theta^2} \times \theta = 76.8$$
$$\frac{46.08}{\theta} = 76.8$$
$$\theta = \frac{46.08}{76.8}$$
$$\theta = 0.6$$

Evaluate r,

$$r = \frac{9.6}{0.6}$$
$$r = 16$$

Now let's construct an equation for the area of shaded region,

$$\text{Area of shaded region } = \text{Area of sector } OAB - \text{Area of triangle } OAB$$
$$\text{Area of shaded region } = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$$

Substitute into the equation and simplify,

$$\text{Area of shaded region } = \frac{1}{2} \times 16^2 \times 0.6 - \frac{1}{2} \times 16^2 \times \sin 0.6$$
$$\text{Area of shaded region } = 76.8 - 72.27423659$$
$$\text{Area of shaded region } = 4.53$$

Therefore, the final answer is,

$$\text{Area of shaded region } = 4.53$$

(b) Find the perimeter of the shaded region.

$$r = 16 \quad \theta = 0.6$$

The perimeter is,

$$\text{Perimeter } = AB + arc\ AB$$

Let's find $arc\ AB$,

$$arc\ AB = r\theta$$
$$arc\ AB = 16 \times 0.6$$
$$arc\ AB = 9.6$$

Divide triangle OAB into two right angled triangles.

Let's use $SOHCAHTOA$ to find AB,

$$\sin\left(\frac{1}{2} \times 0.6\right) = \frac{0.5AB}{16}$$
$$0.5AB = 16 \sin\left(\frac{1}{2} \times 0.6\right)$$
$$AB = 2 \times 16 \sin\left(\frac{1}{2} \times 0.6\right)$$
$$AB = 9.456646613$$

Now let's find the perimeter,

$$\text{Perimeter } = AB + arc\ AB$$

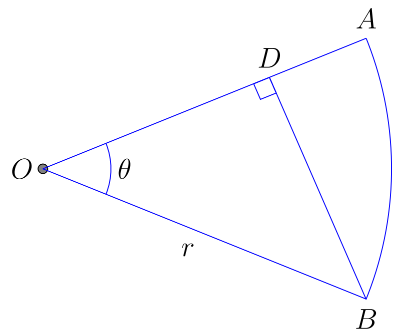
$$\text{Perimeter } = 9.6 + 9.4566$$

$$\text{Perimeter } = 19.1$$

Therefore, the final answer is,

$$\text{Perimeter } = 19.1$$

4.



The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians. (9709/11/M/J/22 number 5)

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r.

$$\theta = \frac{1}{6}\pi$$

Let's construct an equation for the area of BCD,

$$\text{Area of } BCD = \text{Area of sector } ABC - \text{Area of triangle } ABD$$

Let's find the area of sector ABC in terms of r,

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}r^2 \times \frac{1}{6}\pi$$

$$A = \frac{1}{12}\pi r^2$$

$$\text{Area of sector } ABC = \frac{1}{12}\pi r^2$$

Let's find the area of triangle ABD in terms of r, using $SOHCAHTOA$,

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2} \times AD \times BD$$

Let's find AD and BD,

$$\cos\frac{1}{6}\pi = \frac{AD}{r} \quad \sin\frac{1}{6}\pi = \frac{BD}{r}$$
$$\frac{\sqrt{3}}{2} = \frac{AD}{r} \quad \frac{1}{2} = \frac{BD}{r}$$
$$AD = \frac{\sqrt{3}}{2}r \quad BD = \frac{1}{2}r$$

Let's go back to the area of the triangle,

$$A = \frac{1}{2} \times AD \times BD$$

$$A = \frac{1}{2} \times \frac{\sqrt{3}}{2}r \times \frac{1}{2}r$$
$$A = \frac{\sqrt{3}}{8}r^2$$

Now let's go back to the area of BCD,

$$\text{Area of } BCD = \text{Area of sector } ABC - \text{Area of triangle } ABD$$

$$\text{Area of } BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$$

Therefore, the final answer is,

$$\text{Area of } BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$$

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r.

$$BD = \frac{\sqrt{3}}{2}r \quad AD = r\cos\theta$$

Let's construct an expression for the perimeter,

$$\text{Perimeter } = BD + CD + arc\ BC$$

Substitute,

$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + (AC - AD) + r\theta$$
$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + r - r\cos\theta + r\theta$$

Our expression is in terms of r and θ. We need to get rid of θ.

We can use $SOHCAHTOA$ to find the value of θ,

$$\sin\theta = \frac{BD}{AB}$$

$$\sin\theta = \frac{\frac{\sqrt{3}}{2}r}{r}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-}\frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

Let's substitute θ in our expression for perimeter with $\frac{\pi}{3}$,

$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + r - r\cos\theta + r\theta$$

$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + r - r\cos\frac{\pi}{3} + r\frac{\pi}{3}$$

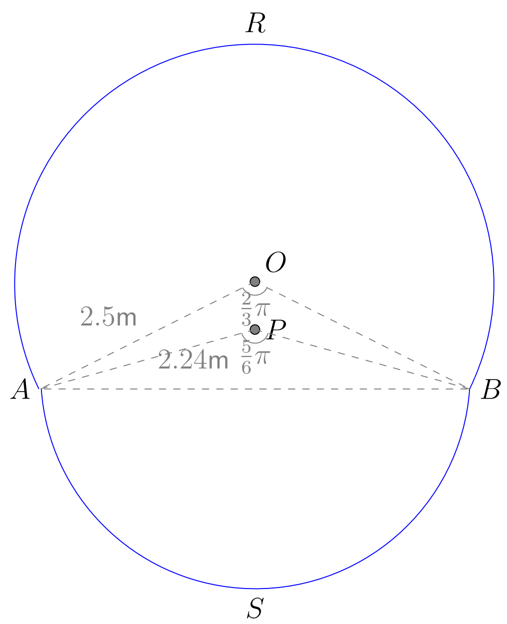
$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + r - \frac{1}{2}r + \frac{\pi}{3}r$$

$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + \frac{1}{2}r + \frac{\pi}{3}r$$

Therefore, the final answer is,

$$\text{Perimeter } = \frac{\sqrt{3}}{2}r + \frac{1}{2}r + \frac{\pi}{3}r$$

5.



The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of a radius 2.5m, with centre O, a sector $PASB$ of another circle of radius 2.24m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$. (9709/12/O/N/22 number 10)

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places.

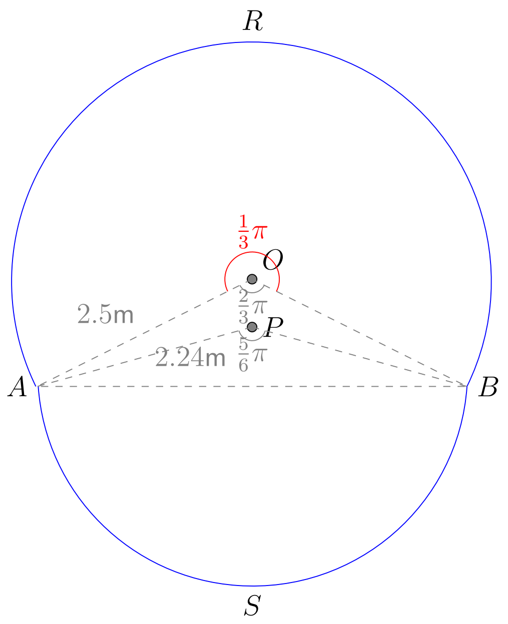
$$r_{OARB} = 2.5 \quad r_{PASB} = 2.24 \quad \theta_{AOB} = \frac{2}{3}\pi \quad \theta_{APB} = \frac{5}{6}\pi$$

Let's construct an expression for the perimeter of the cross-section,

$$\text{Perimeter } = arc\ ARB + arc\ ASB$$

$$\text{Perimeter } = r\theta + r\theta$$

Let's sketch an image of the problem,



Let's substitute into the expression,

$$\text{Perimeter} = 2.5 \times \frac{1}{3}\pi + 2.24 \times \frac{5}{6}\pi$$

$$\text{Perimeter} = \frac{26}{5}\pi$$

$$\text{Perimeter} = 16.34$$

Therefore, the final answer is,

$$\text{Perimeter} = 16.34$$

(b) Find the difference in area of the triangles AOB and APB, giving your answers correct to 2 decimal places.

$$r_{OARB} = 2.5 \quad r_{PASB} = 2.24 \quad \theta_{AOB} = \frac{2}{3}\pi \quad \theta_{APB} = \frac{5}{6}\pi$$

We are required to find the difference in area between the two triangles,

$$\text{Area of triangle } AOB - \text{Area of triangle } APB$$

$$\frac{1}{2}r_{OARB}^2 \sin\theta_{AOB} - \frac{1}{2}r_{PASB}^2 \sin\theta_{APB}$$

Substitute in the values of r and θ,

$$\frac{1}{2} \times (2.5)^2 \times \sin\frac{2}{3}\pi - \frac{1}{2} \times (2.24)^2 \times \sin\frac{5}{6}\pi$$

Simplify,

$$\frac{25\sqrt{3}}{16} - \frac{784}{625}$$
$$1.451929387$$

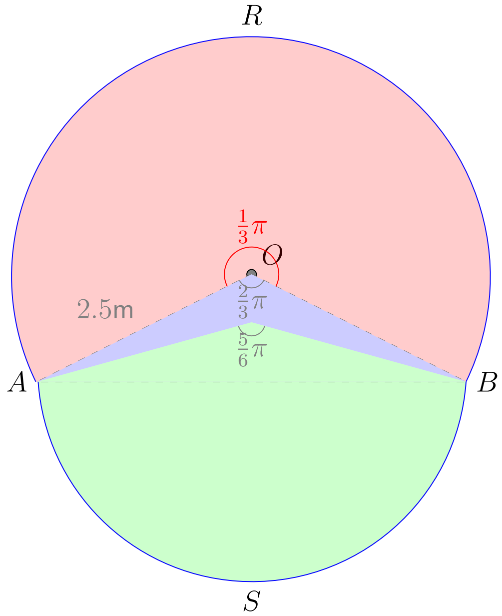
Therefore, the final answer is,

$$1.45$$

(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place.

$$r_{OARB} = 2.5 \quad r_{PASB} = 2.24 \quad \theta_{AOB} = \frac{2}{3}\pi \quad \theta_{APB} = \frac{5}{6}\pi$$

Let's sketch a diagram of the problem,



Let's construct an expression of the area $RASB$,

Area of $RASB$ = Area of sector ARB + Area of difference of two triangles + Area of sector ASB

Let's find the area of sector ARB,

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times (2.5)^2 \times \frac{4}{3}\pi$$

$$A = \frac{25}{6}\pi$$

Now let's find the area of sector ASB,

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times (2.24)^2 \times \frac{5}{6}\pi$$

$$A = \frac{784}{375}\pi$$

The area of the difference of the two triangles we already found in part b to be,

$$A = 1.451929387$$

Let's go back to the expression of the area $RASB$,

Area of $RASB =$ Area of sector $ARB +$ Area of difference of two triangles $+$ Area of sector ASB

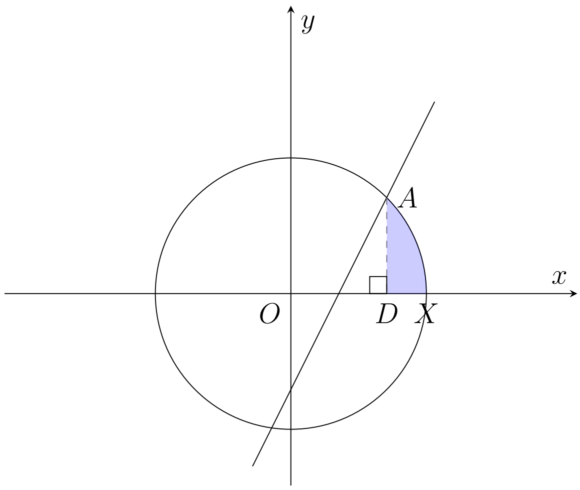
$$\text{Area of } RASB = \frac{25}{6}\pi + \frac{784}{375}\pi + 1.451929387$$

$$\text{Area of } RASB = 21.10992182$$

Therefore, the final answer is,

$$\text{Area of } RASB = 21.1$$

6.



The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points $A(1, 1)$ and B. The point D on the x-axis is such that AD is perpendicular to the x-axis. Find an exact expression for the perimeter of the shaded region. (9709/13/O/N/22 number 10c)

$$x^2 + y^2 = 2 \quad A(1, 1)$$

Let's construct an expression for the perimeter,

$$\text{Perimeter } = AD + DX + arc\ AX$$

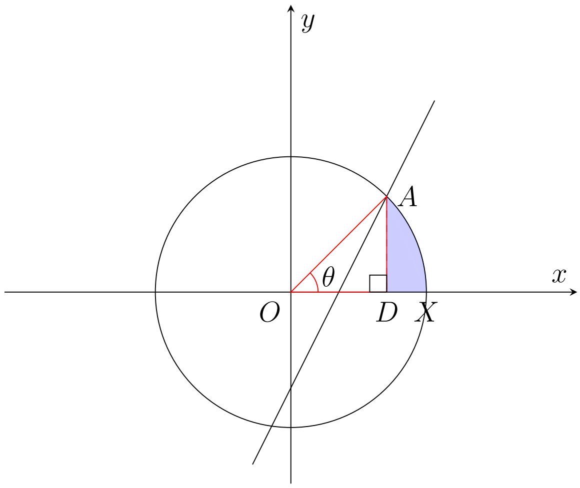
$$\text{Perimeter } = 1 + (OX - OD) + r\theta$$

$$\text{Perimeter } = 1 + (r - 1) + r\theta$$

From the equation of the circle, we can tell that the radius is,

$$r = \sqrt{2}$$

To find θ we will use triangle OAD,



$$\sin\theta = \frac{AD}{OA}$$
$$\sin\theta = \frac{1}{\sqrt{2}}$$
$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$
$$\theta = \frac{\pi}{4}$$

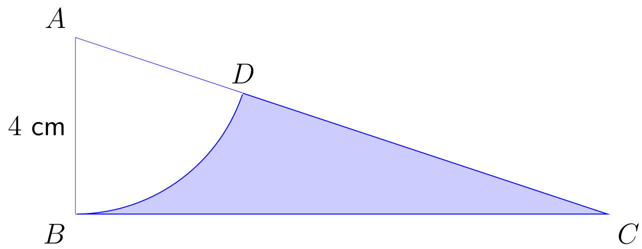
Let's substitute into the expression of the perimeter,

$$\text{Perimeter } = 1 + (r-1) + r\theta$$
$$\text{Perimeter } = 1 + (\sqrt{2}-1) + \sqrt{2} \times \frac{\pi}{4}$$
$$\text{Perimeter } = 1 + \sqrt{2} - 1 + \frac{\sqrt{2}}{4}\pi$$
$$\text{Perimeter } = \sqrt{2} + \frac{\sqrt{2}}{4}\pi$$

Therefore, the final answer is,

$$\text{Perimeter } = \sqrt{2} + \frac{\sqrt{2}}{4}\pi$$

7.



The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$cm. The sector ABD is part of a circle with centre A. The area of the sector is 10cm^2. (9709/13/M/J/21 number 5)

(a) Find angle BAD in radians.

$$AB = 4 \quad A = 10$$

The radius of sector ABD is AB,

$$r = 4$$

We are given that the area of the sector is,

$$A = 10$$

Let's use the formula for area of a sector to find angle BAD,

$$\frac{1}{2}r^2\theta = 10$$

$$\frac{1}{2}(4)^2\theta = 10$$

$$8\theta = 10$$

$$\theta = \frac{5}{4}$$

Therefore, the final answer is,

$$\text{Angle } BAD = \frac{5}{4}$$

(b) Find the perimeter of the shaded region.

$$r = 4 \quad \theta = \frac{5}{4}$$

Let's construct an expression for the perimeter,

$$\text{Perimeter } = BC + CD + arc\ BD$$

$$\text{Perimeter } = BC + (AC - AD) + r\theta$$

$$\text{Perimeter } = BC + (AC - 4) + 4 \times \frac{5}{4}$$

$$\text{Perimeter } = BC + (AC - 4) + 5$$

Let's find BC and AC using $SOHCAHTOA$,

$$\tan\frac{5}{4} = \frac{BC}{4} \quad \cos\frac{5}{4} = \frac{4}{AC}$$

$$BC = 4\tan\frac{5}{4} \quad AC = \frac{4}{\cos\frac{5}{4}}$$

$$BC = 12.0383 \quad AC = 12.6854$$

Let's go back to the expression for perimeter,

$$\text{Perimeter } = BC + (AC - 4) + 5$$

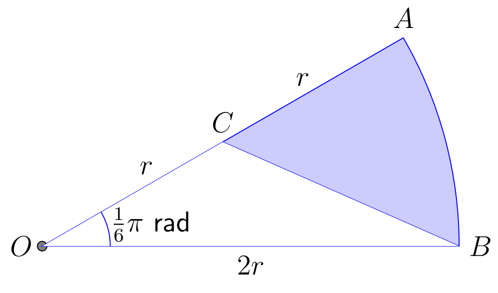
$$\text{Perimeter } = 12.0383 + (12.6854 - 4) + 5$$

$$\text{Perimeter } = 25.7$$

Therefore, the final answer is,

$$\text{Perimeter } = 25.7$$

8.



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA. (9709/12/M/J/20 number 7)

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$.

$$\text{radius } = 2r \quad \theta = \frac{1}{6}\pi$$

Let's use the cosine rule to find BC,

$$BC^2 = OB^2 + OC^2 - 2(OB)(OC)\cos\theta$$

Substitute,

$$BC^2 = (2r)^2 + r^2 - 2(2r)(r)\cos\frac{1}{6}\pi$$

Simplify,

$$BC^2 = 4r^2 + r^2 - 4r^2 \times \frac{\sqrt{3}}{2}$$

$$BC^2 = 5r^2 - 2\sqrt{3}r^2$$

Factor out r^2,

$$BC^2 = r^2\left(5 - 2\sqrt{3}\right)$$

Take the square root of both sides,

$$BC = \sqrt{r^2\left(5 - 2\sqrt{3}\right)}$$

Distribute the square root,

$$BC = \sqrt{r^2} \times \sqrt{5 - 2\sqrt{3}}$$

$$BC = r\sqrt{5 - 2\sqrt{3}}$$

Therefore, the final answer is,

$$BC = r\sqrt{5 - 2\sqrt{3}}$$

(b) Find the exact perimeter of the shaded region.

$$\text{radius } = 2r \quad \theta = \frac{1}{6}\pi \quad BC = r\sqrt{5 - 2\sqrt{3}}$$

Let's construct an expression of the perimeter,

$$\text{Perimeter } = BC + AC + arc\ AB$$

Substitute and simplify,

$$\text{Perimeter } = r\sqrt{5 - 2\sqrt{3}} + r + 2r \times \frac{1}{6}\pi$$

$$\text{Perimeter } = r\sqrt{5 - 2\sqrt{3}} + r + \frac{1}{3}\pi r$$

Therefore, the final answer is,

$$\text{Perimeter } = r\sqrt{5-2\sqrt{3}} + r + \frac{1}{3}\pi r$$

(c) Find the exact area of the shaded region.

$$\text{radius } = 2r \quad \theta = \frac{1}{6}\pi \quad OB = 2r \quad OC = r$$

Let's construct an expression for the area of the shaded region,

$$\text{Area of shaded region } = \text{Area of sector } OAB - \text{Area of triangle } OBC$$

Let's find the area of the sector OAB,

$$A = \frac{1}{2} \times (2r)^2 \times \frac{1}{6}\pi$$

$$\text{Area of sector } OAB = \frac{1}{3}\pi r^2$$

Let's find the area of triangle OBC,

$$A = \frac{1}{2} \times OC \times OB \times \sin\theta$$

$$A = \frac{1}{2} \times r \times 2r \times \sin\frac{1}{6}\pi$$

$$A = r^2 \times \frac{1}{2}$$

$$\text{Area of triangle } OBC = \frac{1}{2}r^2$$

Let's go back to our expression of the area of the shaded region,

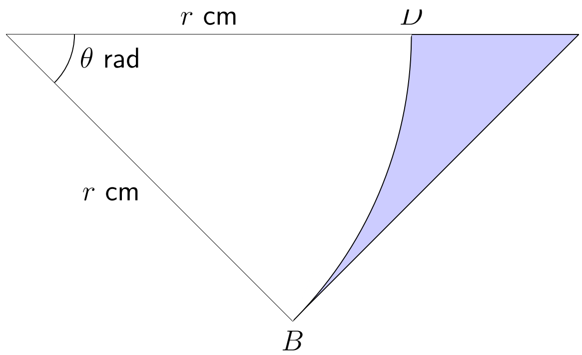
$$\text{Area of shaded region } = \text{Area of sector } OAB - \text{Area of triangle } OBC$$

$$\text{Area of shaded region } = \frac{1}{3}\pi r^2 - \frac{1}{2}r^2$$

Therefore, the final answer is,

$$\text{Area of shaded region } = \frac{1}{3}\pi r^2 - \frac{1}{2}r^2$$

9.



In the diagram, ABC is an isosceles triangle with $AB = BC = r$cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector a circle with centre A. (9709/12/O/N/20 number 8)

(a) Express the area of the shaded region in terms of r and θ.

Let's construct an expression for the area of the shaded region,

$$\text{Area of shaded region } = \text{Area of triangle } ABC - \text{Area of sector } ABD$$

Let's find the area of triangle ABC. We will use the formula,

$$A = \frac{1}{2}(AB)(BC)\sin ABC$$

$$A = \frac{1}{2}r^2 \sin ABC$$

Since ABC is an isosceles triangle and the angles within a triangle add up to 180 or π,

$$ABC = \pi - 2\theta$$

Substitute into the area of the triangle,

$$\text{Area of triangle } ABC = \frac{1}{2}r^2 \sin(\pi - 2\theta)$$

Let's find the area of sector ABD,

$$\text{Area of sector } ABD = \frac{1}{2}r^2\theta$$

Go back to the expression for the area of the shaded region,

$$\text{Area of shaded region } = \text{Area of triangle } ABC - \text{Area of sector } ABD$$

$$\text{Area of shaded region } = \frac{1}{2}r^2 \sin(\pi - 2\theta) - \frac{1}{2}r^2\theta$$

Therefore, the final answer is,

$$\text{Area of shaded region } = \frac{1}{2}r^2\sin(\pi - 2\theta) - \frac{1}{2}r^2\theta$$

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region.

$$r = 10 \quad \theta = 0.6$$

Let's construct an expression for the perimeter of the shaded region,

$$\text{Perimeter } = BC + CD + arc\ BD$$

$$\text{Perimeter } = r + (AC - AD) + r\theta$$

$$\text{Perimeter } = r + (AC - r) + r\theta$$

Let's use the sine rule to find AC,

$$\frac{AC}{\sin ABC} = \frac{BC}{\sin BAD}$$

$$\frac{AC}{\sin(\pi - 2\theta)} = \frac{r}{\sin\theta}$$

$$\frac{AC}{\sin(\pi - 2(0.6))} = \frac{10}{\sin 0.6}$$

$$AC = \frac{10}{\sin 0.6} \times \sin(\pi - 2(0.6))$$

$$AC = 16.5067$$

Now let's go back to the expression for the perimter of the shaded region,

$$\text{Perimeter } = r + (AC - r) + r\theta$$

$$\text{Perimeter } = 10 + (16.5067 - 10) + 10 \times 0.6$$

$$\text{Perimeter } = 22.5$$

Therefore, the final answer is,

$$\text{Perimeter } = 22.5$$