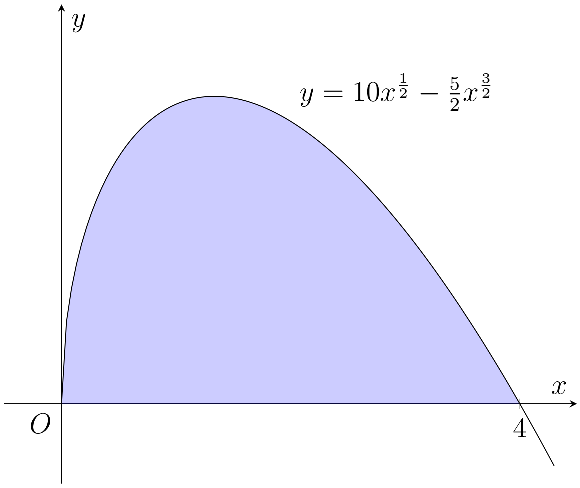
Pure Maths 1

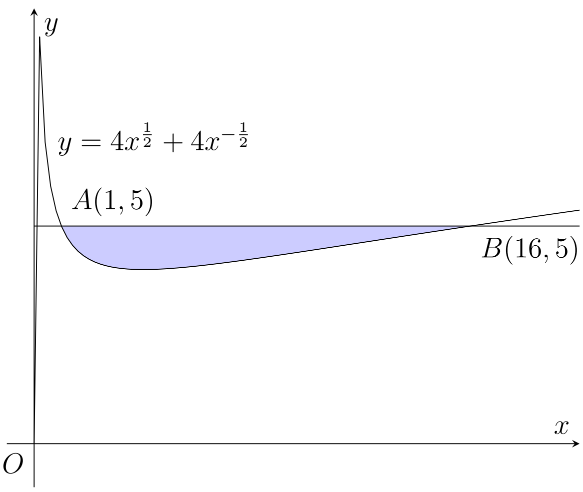
1.8 Integration - Easy

GCEALevelMaths9709

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Pure Mathematics 1**
Topic:	**1.8 Integration**
Difficulty:	**Easy**

Questions

1. At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} - 2$. (9709/12/F/M/23 number 10b) Find the equation of the curve.
2. The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 24$. The curve has a stationary point at $(4, -15)$. (9709/11/M/J/23 number 11c) Find the equation of the curve.
3. The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point $(4, 5)$. (9709/12/M/J/23 number 1) Find the equation of the curve.
4. 

   The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$. (9709/12/M/J/23 number 5) Find the area of the shaded region.
5. A curve which passes through $(0, 3)$ has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$. (9709/13/M/J/23 number 9a) Find the equation of the curve.
6. A curve with equation $y = f(x)$ is such that $f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $f(8) = 5$. (9709/12/F/M/22 number 1) Find $f(x)$.
7. The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$. (9709/12/M/J/22 number 3) Find the equation of the curve.
8. 

The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1, 5)$ and $B(16, 5)$. (9709/13/M/J/22 number 8b) Calculate the area of the shaded region.

9. The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$. (9709/13/M/J/22 number 10a) Find $\int_1^{\infty} f(x)\ dx$

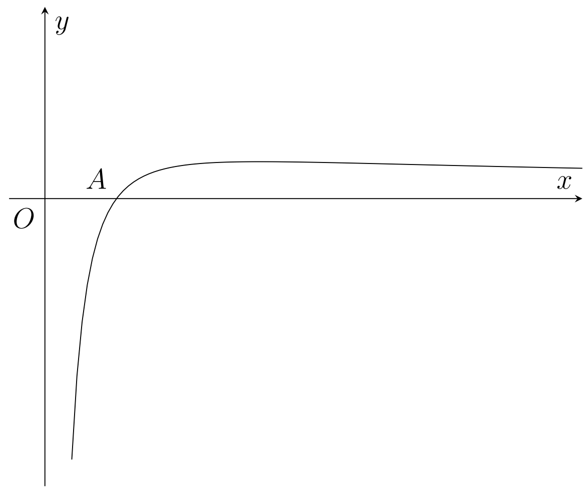
10. The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P(6, 4)$. (9709/11/O/N/22 number 2b) Find the equation of the curve.

11. The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point $(3, 5)$. (9709/12/O/N/22 number 8a) Find the equation of the curve.

12. The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$. (9709/13/O/N/22 number 7b) Find $f(x)$ given that the curve passes through the point $(-1, 5)$.

13. A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A(1, -3)$ lies on the curve. (9709/12/F/M/21 number 6b) Find the equation of the curve.

14.



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point $A(4, 0)$. (9709/12/F/M/21 number 11d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$.

15. The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $\left(\frac{1}{2}, 4\right)$. (9709/11/M/J/21 number 1) Find the equation of the curve.

16. The equation of a curve is $y = 2\sqrt{3x + 4} - x$. (9709/11/M/J/21 number 11d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$.

17. A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$. (9709/13/M/J/21 number 1) Find $f(x)$.

Answers

1. At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} - 2$. (9709/12/F/M/23 number 10b) Find the equation of the curve.

$$\frac{dy}{dx} = x^{-\frac{1}{2}} - 2$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 2x + c$$
$$y = 2x^{\frac{1}{2}} - 2x + c$$

The curve passes through $(4, -1)$. Let's use this point to evaluate c,

$$-1 = 2(4)^{\frac{1}{2}} - 2(4) + c$$
$$-1 = 4 - 8 + c$$
$$-1 = -4 + c$$
$$c = 3$$

Therefore, the equation of the curve is,

$$y = 2x^{\frac{1}{2}} - 2x + 3$$

2. The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 24$. The curve has a stationary point at $(4, -15)$. (9709/11/M/J/23 number 11c) Find the equation of the curve.

$$\frac{dy}{dx} = 6x^2 - 30x + 24$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{6x^3}{3} - \frac{30x^2}{2} + 24x + c$$
$$y = 2x^3 - 15x^2 + 24x + c$$

The curve passes through $(4, -15)$. Let's use this point to evaluate c,

$$-15 = 2(4)^3 - 15(4)^2 + 24(4) + c$$
$$-15 = 128 - 240 + 96 + c$$
$$-15 = -16 + c$$
$$c = 1$$

Therefore, the equation of the curve is,

$$y = 2x^3 - 15x^2 + 24x + 1$$

3. The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point $(4, 5)$. (9709/12/M/J/23 number 1) Find the equation of the curve.

$$\frac{dy}{dx} = 4(x-3)^{-3}$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{4(x-3)^{-2}}{-2} + c$$
$$y = -2(x-3)^{-2} + c$$

The curve passes through $(4, 5)$. Let's use this point to evaluate c,

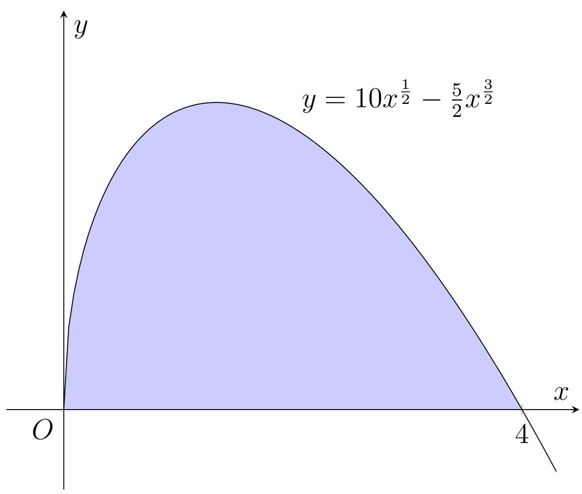
$$5 = -2(4-3)^{-2} + c$$

$$5 = -2 + c$$
$$c = 7$$

Therefore, the equation of the curve is,

$$y = -2(x-3)^{-2} + 7$$

4.



The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points $(0, 0)$ and $(4, 0)$. (9709/12/M/J/23 number 5) Find the area of the shaded region.

$$y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$$

Let's integrate the curve from 0 to 4,

$$\int_0^4 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}\ dx$$

$$\left[\frac{10x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{5x^{\frac{5}{2}}}{2 \times \frac{5}{2}}\right]_0^4$$

$$\left[\frac{20}{3}x^{\frac{3}{2}} - x^{\frac{5}{2}}\right]_0^4$$

Substitute in the limits,

$$\left[\left(\frac{20}{3}(4)^{\frac{3}{2}} - (4)^{\frac{5}{2}}\right) - \left(\frac{20}{3}(0)^{\frac{3}{2}} - (0)^{\frac{5}{2}}\right)\right]$$

$$\left[\frac{64}{3} - 0\right]$$

$$\frac{64}{3}$$

Therefore, the area of the shaded region is,

$$\frac{64}{3}$$

5. A curve which passes through $(0, 3)$ has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$. (9709/13/M/J/23 number 9a) Find the equation of the curve.

$$\frac{dy}{dx} = 1 - 2(x-1)^{-3}$$

Let's integrate $\frac{dy}{dx}$,

$$y = x - \frac{2(x-1)^{-2}}{-2} + c$$

$$y = x + (x-1)^{-2} + c$$

The curve passes through $(0, 3)$. Let's use this point to evaluate c,

$$3 = 0 + (0-1)^{-2} + c$$

$$3 = 1 + c$$

$$c = 2$$

Therefore, the equation of the curve is,

$$y = x + (x-1)^{-2} + 2$$

6. A curve with equation $y = f(x)$ is such that $f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $f(8) = 5$. (9709/12/F/M/22 number 1) Find $f(x)$.

$$f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$$

Let's integrate $f'(x)$,

$$y = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$y = 3x^{\frac{2}{3}} - \frac{3}{4}x^{\frac{4}{3}} + c$$

The curve passes through $(8, 5)$. Let's use this point to evaluate c,

$$5 = 3(8)^{\frac{2}{3}} - \frac{3}{4}(8)^{\frac{4}{3}} + c$$

$$5 = 12 - 12 + c$$

$$c = 5$$

Therefore, the equation of the curve is,

$$f(x) = 3x^{\frac{2}{3}} - \frac{3}{4}x^{\frac{4}{3}} + 5$$

7. The equation of a curve is such that $\frac{dy}{dx} = 3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$. (9709/12/M/J/22 number 3) Find the equation of the curve.

$$\frac{dy}{dx} = 3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2} \times 4} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$$

The curve passes through $\left(4, \frac{5}{2}\right)$. Let's use this point to evaluate c,

$$\frac{5}{2} = \frac{1}{2}(4(4)-7)^{\frac{3}{2}} - 8(4)^{\frac{1}{2}} + c$$

$$\frac{5}{2} = \frac{27}{2} - 16 + c$$

$$\frac{5}{2} = -\frac{5}{2} + c$$

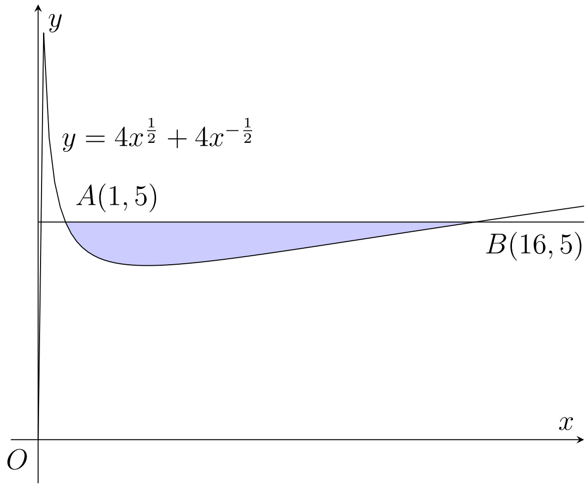
$$c = \frac{5}{2} + \frac{5}{2}$$

$$c = 5$$

Therefore, the equation of the curve is,

$$y = \frac{1}{2}(4x-7)^{\frac{3}{2}} - 8x^{\frac{1}{2}} + 5$$

8.



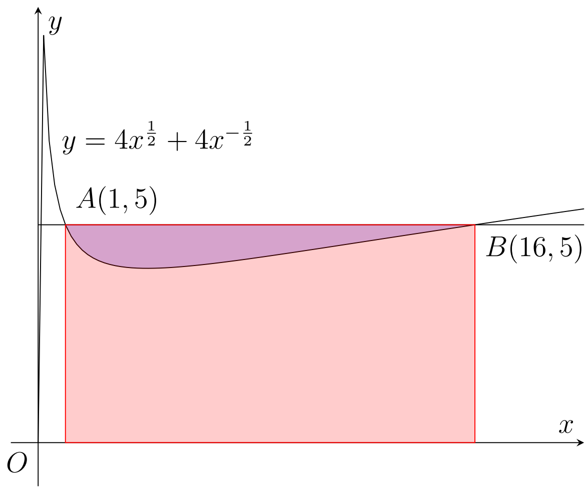
The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A(1,5)$ and $B(16,5)$. (9709/13/M/J/22 number 8b) Calculate the area of the shaded region.

$$y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$$

Let's construct an expression for the area of the shaded region,

$$\text{Area of shaded region } = \text{Area under the line} - \text{Area under the curve}$$

Let's find the area under the line,



The area under the line forms a rectangle,

$$\text{Area under the line} = 15 \times 5$$

$$\text{Area under the line} = 75$$

Now let's find the area under the curve,

$$y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$$

Integrate the curve from 1 to 16,

$$\int_1^{16} x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\ dx$$

$$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{16}$$

$$\left[\frac{2}{3}x^{\frac{3}{2}} + 8x^{\frac{1}{2}}\right]_1^{16}$$

Substitute in the limits,

$$\left[\left(\frac{2}{3}(16)^{\frac{3}{2}} + 8(16)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(1)^{\frac{3}{2}} + 8(1)^{\frac{1}{2}}\right)\right]$$

$$\left[\frac{224}{3} - \frac{26}{3}\right]$$

$$\text{Area under the curve} = 66$$

Now let's go back to our expression for the area of the shaded region,

$$\text{Area of shaded region } = \text{Area under the line} - \text{Area under the curve}$$

$$\text{Area of shaded region } = 75 - 66$$

$$\text{Area of shaded region } = 9$$

Therefore, the final answer is,

$$\text{Area of shaded region } = 9$$

9. The function f is defined by $f(x) = (4x+2)^{-2}$ for $x > -\frac{1}{2}$. (9709/13/M/J/22 number 10a) Find $\int_1^{\infty} f(x)\ dx$

$$\int_1^{\infty} (4x+2)^{-2}\ dx$$

Let's integrate $f(x)$ from 1 to ∞,

$$\left[\frac{(4x+2)^{-1}}{-1 \times 4}\right]_1^{\infty}$$

$$\left[-\frac{1}{4}(4x+2)^{-1}\right]_1^{\infty}$$

$$\left[-\frac{1}{4(4x+2)}\right]_1^{\infty}$$

Substitute in the limits,

$$\left[\left(-\frac{1}{4(4(\infty)+2)}\right)-\left(-\frac{1}{4(4(1)+2)}\right)\right]$$

$$\left[0-\left(-\frac{1}{24}\right)\right]$$

Note: $\frac{1}{\infty}$ **is so small that it is assigned a value of** 0**. You can try dividing** 1 **by a very large number on your calculator and you'll notice that it is very close to** 0**, e.g** $\frac{1}{100000}=0.00001$

$$\left[0+\frac{1}{24}\right]$$

$$\frac{1}{24}$$

Therefore, the final answer is,

$$\int_{1}^{\infty} f(x)\ dx=\frac{1}{24}$$

10. The equation of a curve is such that $\frac{dy}{dx}=12\left(\frac{1}{2}x-1\right)^{-4}$. It is given that the curve passes through the point $P(6,4)$. (9709/11/O/N/22 number 2b) Find the equation of the curve.

$$\frac{dy}{dx}=12\left(\frac{1}{2}x-1\right)^{-4}$$

Let's integrate $\frac{dy}{dx}$**,**

$$y=\frac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3\times\frac{1}{2}}+c$$

$$y=-8\left(\frac{1}{2}x-1\right)^{-3}+c$$

The curve passes through $P\,(6,4)$**. Let's use this point to evaluate** c**,**

$$4=-8\left(\frac{1}{2}(6)-1\right)^{-3}+c$$

$$4=-1+c$$

$$c=5$$

Therefore, the equation of the curve is,

$$y=-8\left(\frac{1}{2}x-1\right)^{-3}+5$$

11. The equation of a curve is such that $\frac{dy}{dx}=3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}$. The curve passes through the point $(3,5)$. (9709/12/O/N/22 number 8a) Find the equation of the curve.

$$\frac{dy}{dx}=3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$$

The curve passes through $(3,5)$. Let's use this point to evaluate c,

$$5 = 2(3)^{\frac{3}{2}} - 6(3)^{\frac{1}{2}} + c$$

$$5 = 0 + c$$

$$c = 5$$

Therefore, the equation of the curve is,

$$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 5$$

12. The curve $y = f(x)$ is such that $f'(x) = \frac{-3}{(x+2)^4}$. (9709/13/O/N/22 number 7b) Find $f(x)$ given that the curve passes through the point $(-1,5)$.

$$f'(x) = -3(x+2)^{-4}$$

Let's integrate $f'(x)$,

$$y = \frac{-3(x+2)^{-3}}{-3} + c$$

$$y = (x+2)^{-3} + c$$

The curve passes through $(-1,5)$. Let's use this point to evaluate c,

$$5 = (-1+2)^{-3} + c$$

$$5 = 1 + c$$

$$c = 4$$

Therefore, the equation of the curve is,

$$f(x) = (x+2)^{-3} + 4$$

13. A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A(1,-3)$ lies on the curve. (9709/12/F/M/21 number 6b) Find the equation of the curve.

$$\frac{dy}{dx} = 6(3x-2)^{-3}$$

Let's integrate $\frac{dy}{dx}$,

$$y = \frac{6(3x-2)^{-2}}{-2 \times 3} + c$$

$$y = -(3x-2)^{-2} + c$$

The curve passes through $A\,(1,-3)$. Let's use this point to evaluate c,

$$-3 = -(3(1)-2)^{-2} + c$$

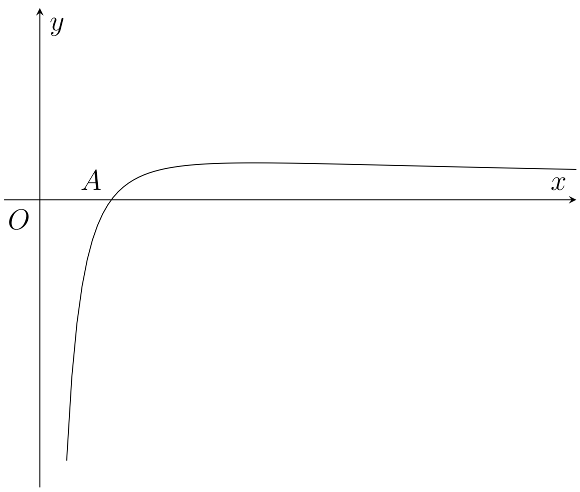
$$-3 = -1 + c$$

$$c = -2$$

Therefore, the equation of the curve is,

$$y = -(3x-2)^{-2} - 2$$

14.



The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point $A(4,0)$. (9709/12/F/M/21 number 11d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$.

$$y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$$

Integrate the curve from 4 to 9,

$$\int_4^9 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)\,dx$$

$$9\int_4^9 \left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)\,dx$$

$$9\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right]_4^9$$

$$9\left[2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}}\right]_4^9$$

Substitute in the limits,

$$9\left[\left(2(9)^{\frac{1}{2}}+8(9)^{-\frac{1}{2}}\right)-\left(2(9)^{\frac{1}{2}}+8(9)^{-\frac{1}{2}}\right)\right]$$

$$9\left[\frac{26}{3}-8\right]$$

$$9\left(\frac{2}{3}\right)$$

$$6$$

Therefore, the area of the required region is,

$$6$$

15. The equation of a curve is such that $\frac{dy}{dx}=\frac{3}{x^4}+32x^3$. It is given that the curve passes through the point $\left(\frac{1}{2},4\right)$. (9709/11/M/J/21 number 1) Find the equation of the curve.

$$\frac{dy}{dx}=3x^{-4}+32x^3$$

Let's integrate $\frac{dy}{dx}$,

$$y=\frac{3x^{-3}}{-3}+\frac{32x^4}{4}+c$$

$$y=-x^{-3}+8x^4+c$$

The curve passes through $\left(\frac{1}{2},4\right)$. Let's use this point to evaluate c,

$$4=-\left(\frac{1}{2}\right)^{-3}+8\left(\frac{1}{2}\right)^4+c$$

$$4=-8+\frac{1}{2}+c$$

$$4=-\frac{15}{2}+c$$

$$c=4+\frac{15}{2}$$

$$c=\frac{23}{2}$$

Therefore, the equation of the curve is,

$$y=-x^{-3}+8x^4+\frac{23}{2}$$

16. The equation of a curve is $y=2\sqrt{3x+4}-x$. (9709/11/M/J/21 number 11d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x=0$ and $x=4$.

$$y=2(3x+4)^{\frac{1}{2}}-x$$

Let's integrate the curve from 0 to 4,

$$\int_0^4 2(3x+4)^{\frac{1}{2}} - x \; dx$$

$$\left[\frac{2(3x+4)^{\frac{3}{2}}}{\frac{3}{2}\times 3} - \frac{x^2}{2}\right]_0^4$$

$$\left[\frac{4}{9}(3x+4)^{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$$

Substitute in the limits,

$$\left[\left(\frac{4}{9}(3(4)+4)^{\frac{3}{2}} - \frac{(4)^2}{2}\right) - \left(\frac{4}{9}(3(0)+4)^{\frac{3}{2}} - \frac{(0)^2}{2}\right)\right]$$

$$\left[\frac{184}{9} - \frac{32}{9}\right]$$

$$\frac{152}{9}$$

Therefore, the exact area of the required region is,

$$\frac{152}{9}$$

17. A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$. (9709/13/M/J/21 number 1) Find $f(x)$.

$$f'(x) = 6x^2 - 8x^{-2}$$

Let's integrate $f'(x)$,

$$y = \frac{6x^3}{3} - \frac{8x^{-1}}{-1} + c$$

$$y = 2x^3 + 8x^{-1} + c$$

The curve passes through $(2, 7)$. Let's use this point to evaluate c,

$$7 = 2(2)^3 + 8(2)^{-1} + c$$

$$7 = 16 + 4 + c$$

$$7 = 20 + c$$

$$c = -13$$

Therefore, the equation of the curve is,

$$f(x) = 2x^3 + 8x^{-1} - 13$$