Probability and Statistics 1

5.3 Probability - Easy

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Probability and Statistics 1**
Topic:	**5.3 Probability**
Difficulty:	**Easy**

Questions

1. A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table. (9709/52/M/J/20 number 2)

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

   (a) Find the probability that a randomly chosen student is at Canton college and prefers hockey.

   (b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer.

   (c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer.

2. Two ordinary fair dice, one red and the other blue, are thrown. Event A is 'the score on the red die is divisible by 3'. Event B is 'the sum of the two scores is at least 9'. (9709/51/O/N/20 number 1)

   (a) Find $P(A \cap B)$.

   (b) Hence determine whether or not the events A and B are independent.

3. Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3. (9709/52/F/M/21 number 2)

   (a) Find the probability that on a randomly chosen day Georgie wears a hat.

   (b) Find the probability that on a randomly chosen Georgie wears a yellow scarf given that she does not wear a hat.

4. There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table. (9709/52/F/M/21 number 7a)

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

   A student is chosen at random

   (a) Find the probability that the student prefers swimming.

   (b) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer.

5. In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the table below. (9709/53/M/J/21 number 7a)

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) Find the probability that a randomly chosen household is in Shan and has poor broadband service.

(b) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan.

6. Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table. (9709/52/O/N/21 number 1)

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar.

(b) Find the probability that the student is male given that he plays the drums.

(c) Determine whether the events 'the student plays the guitar' and 'the student is female' are independent, justifying your answer.

7. On any day, Kino travels to school by bus, by car or on foot with the probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25. (9709/52/O/N/22 number 1) The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x.

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late.

8. Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time. (9709/52/O/N/22 number 5ab) Events A and B are defined as follows. A: all three coins show the same result B: at least one of the biased coin shows a head

(a) Show that $P(B) = \frac{7}{16}$.

(b) Find $P(A|B)$.

9. The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day if it is not raining and Aran is not wearing a hat or a scarf is 0.36. (9709/52/F/M/23 number 4) Find the value of x.

Answers

1. A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table. (9709/52/M/J/20 number 2)

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey.

Identify the cell that contains Canton college and hockey,

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

This means that our probability is,

$$\frac{56}{500}$$

This simplifies to give,

$$\frac{14}{125}$$

Therefore, the final answer is,

$$\frac{14}{125}$$

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer.

This is conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A - student prefers soccer

B - student is at Devar college

A $\cap$ B - student prefers soccer and is at Devar college

Let's find $P(A \cap B)$, identify the cell that links soccer to Devar college,

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

Our probability is,

$$P(A \cap B) = \frac{120}{500}$$
$$P(A \cap B) = \frac{6}{25}$$

Now let's find $P(A)$, identify the cell with the total for soccer,

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

Our probability is,

$$P(A) = \frac{280}{500}$$
$$P(A) = \frac{14}{25}$$

Substitute into the formula for conditional probability,

$$P(B|A) = \frac{6}{25} \div \frac{14}{25}$$
$$P(B|A) = \frac{3}{7}$$

Therefore, the final answer is,

$$P(B|A) = \frac{3}{7}$$

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer.

Let's start by defining the events,

X - the student prefers hockey

Y - the student is at Amos or Benn college

If events A and B are independent then,

$$P(X \cap Y) = P(X) \times P(Y)$$

The probability that a student prefers hockey is,

$$P(X) = \frac{220}{500}$$
$$P(X) = \frac{11}{25}$$

The probability that a student is at Amos or Benn college is,

$$P(Y) = \frac{86 + 156}{500}$$
$$P(Y) = \frac{121}{250}$$

The probability that a student prefers hockey and is at Amos or Benn college, identify the cell linking the two,

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

Hence our probability is,

$$P(X \cap Y) = \frac{32 + 72}{500}$$
$$P(X \cap Y) = \frac{26}{125}$$

Now let's check if the events are independent,

$$P(X \cap Y) = P(X) \times P(Y)$$
$$\frac{26}{125} \neq \frac{11}{25} \times \frac{121}{250}$$
$$\frac{26}{125} \neq \frac{1331}{6250}$$

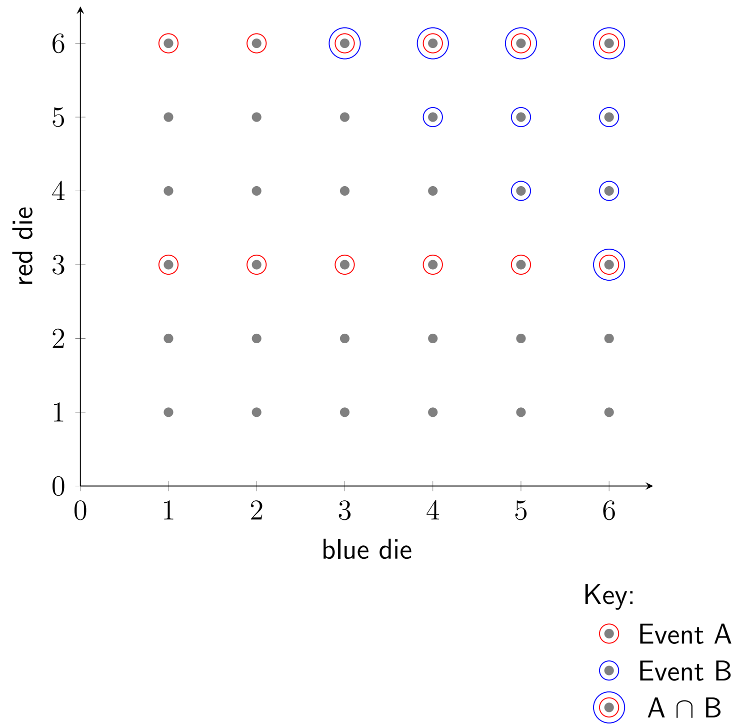
Therefore, the final answer is,

Events A and B are not independent

2. Two ordinary fair dice, one red and the other blue, are thrown. Event A is 'the score on the red die is divisible by 3'. Event B is 'the sum of the two scores is at least 9'. (9709/51/O/N/20 number 1)

   (a) Find $P(A \cap B)$.

Let's draw a possibility space diagram,



From the diagram, we can tell that,

$$P(A \cap B) = \frac{5}{36}$$

Therefore, the final answer is,

$$P(A \cap B) = \frac{5}{36}$$

(b) Hence determine whether or not the events A and B are independent.

If events A and B are independent,

$$P(A \cap B) = P(A) \times P(B)$$

From our diagram we can tell that,

$$P(A) = \frac{12}{36} \quad P(B) = \frac{10}{36}$$

Now let's check if the events are independent,

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{5}{36} \neq \frac{12}{36} \times \frac{10}{36}$$

$$\frac{5}{36} \neq \frac{5}{54}$$

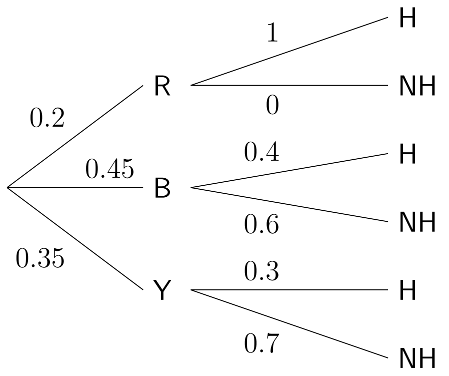
Therefore, the final answer is,

Events A and B are not independent

3. Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3. (9709/52/F/M/21 number 2)

   (a) Find the probability that on a randomly chosen day Georgie wears a hat.

   Let's draw a tree diagram to represent the given information,



   Now let's construct an expression for our problem,

$$P(H) = P(RH) + P(BH) + P(YH)$$

   Read the respective probabilities from the tree diagram and substitute,

$$P(H) = (0.2 \times 1) + (0.45 \times 0.4) + (0.35 \times 0.3)$$

   Simplify,

$$P(H) = 0.485$$

   Therefore, the final answer is,

$$P(H) = 0.485$$

   (b) Find the probability that on a randomly chosen Georgie wears a yellow scarf given that she does not wear a hat.

   This is conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A - She does not wear a hat

B - She wears a yellow scarf

A ∩ B - She does not wear a hat and she wears a yellow scarf

Let's find the probability that she does not wear a hat. In part (a) we found the probability of Georgie wearing a hat to be,

$$P(H) = 0.485$$

We can use that to find $P(A)$,

$$P(A) = 1 - 0.485$$

$$P(A) = 0.515$$

Now let's find the probability that Georgie does not wear a hat and she wears a yellow scarf, you can read that off the tree diagram in part (a),

$$P(A \cap B) = 0.35 \times 0.7$$

$$P(A \cap B) = 0.245$$

Now let's substitute into the formula for conditional probability,

$$P(B|A) = \frac{0.245}{0.515}$$

This simplifies to give,

$$P(B|A) = \frac{49}{103}$$

Therefore, the final answer is,

$$P(B|A) = \frac{49}{103}$$

4. There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table. (9709/52/F/M/21 number 7a)

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random

(a) Find the probability that the student prefers swimming.

These are the cells that represent swimming,

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

This means our probability is,

$$\frac{104 + 31}{400}$$

This simplifies to give,

$$\frac{27}{80}$$

Therefore, the final answer is,

$$\frac{27}{80}$$

(b) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer.

Let's define the events,

A - the student is male

B - the student prefers swimming

A ∩ B - the student is male and prefers swimming

If events A and B are independent,

$$P(A \cap B) = P(A) \times P(B)$$

Add up the tallies for male students to get $P(A)$,

$$P(A) = \frac{31 + 57 + 92}{400}$$

$$P(A) = \frac{9}{20}$$

We already calculated the probability that student prefers swimming in part (a),

$$P(B) = \frac{27}{80}$$

Let's find the probability that the student is male and prefers swimming, identify the correct cell,

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

Our probability is,

$$P(A \cap B) = \frac{31}{400}$$

Now let's check if the events are independent,

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{31}{400} \neq \frac{9}{20} \times \frac{27}{80}$$

$$\frac{31}{400} \neq \frac{243}{1600}$$

Therefore, the final answer is,

Events A and B are not independent

5. In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the table below. (9709/53/M/J/21 number 7a)

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) Find the probability that a randomly chosen household is in Shan and has poor broadband service.

Identify the cell that links Shan to poor broadband service,

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

This means our probability is,

$$\frac{40}{800}$$

This simplifies to give,

$$\frac{1}{20}$$

Therefore, the final answer is,

$$\frac{1}{20}$$

(b) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan.

This is conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A - household is in Shan

B - household has good broadband service

A ∩ B - household is in Shan and has good broadband service

Let's find the probability that the household is in Shan,

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

$$P(A) = \frac{223 + 177 + 40}{800}$$

$$P(A) = \frac{11}{20}$$

Now let's find the probability that the household is in Shan and has good broadband service,

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

$$P(A \cap B) = \frac{177}{800}$$

Substitute into the formula for conditional probability,

$$P(B|A) = \frac{177}{800} \div \frac{11}{20}$$

$$P(B|A) = \frac{177}{440}$$

Therefore, the final answer is,

$$P(B|A) = \frac{177}{440}$$

6. Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table. (9709/52/O/N/21 number 1)

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar.

Identify the cells for guitar,

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

This means our probability is,

$$\frac{44+38}{180}$$

This simplifies to give,

$$\frac{41}{90}$$

Therefore, the final answer is,

$$\frac{41}{90}$$

(b) Find the probability that the student is male given that he plays the drums.

This is conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A - student plays the drums

B - student is male

A ∩ B - student plays the drums and is male

Let's find the probability that the student plays the drums,

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

$$P(A) = \frac{11+20}{180}$$

$$P(A) = \frac{31}{180}$$

Now let's find the probability that the student plays the drums and is male,

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

$$P(A \cap B) = \frac{11}{180}$$

Now let's substitute into the formula for conditional probability,

$$P(B|A) = \frac{11}{180} \times \frac{31}{180}$$

This simplifies to give,

$$P(B|A) = \frac{11}{31}$$

Therefore, the final answer is,

$$P(B|A) = \frac{11}{31}$$

(c) Determine whether the events 'the student plays the guitar' and 'the student is female' are independent, justifying your answer.

Let's define the events,

A - the student plays the guitar

B - the student is female

A ∩ B - the student plays the guitar and is female

If events A and B are independent,

$$P(A \cap B) = P(A) \times P(B)$$

Add up the tallies for guitar students to get $P(A)$,

$$P(A) = \frac{44 + 38}{180}$$

$$P(A) = \frac{41}{90}$$

Add up the tallies for female students to get $P(B)$,

$$P(B) = \frac{42 + 38 + 20}{180}$$

$$P(B) = \frac{5}{9}$$

Let's find the probability that the student plays the guitar and is female, identify the correct cell,

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

Our probability is,

$$P(A \cap B) = \frac{38}{180}$$

$$P(A \cap B) = \frac{19}{90}$$

Now let's check if the events are independent,

$$P(A \cap B) = P(A) \times P(B)$$

$$\frac{19}{90} \neq \frac{41}{90} \times \frac{5}{9}$$

$$\frac{19}{90} \neq \frac{41}{162}$$

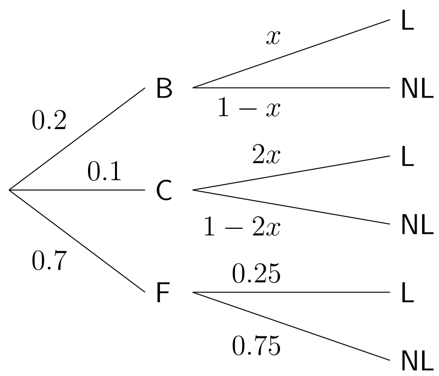
Therefore, the final answer is,

Events A and B are not independent

7. On any day, Kino travels to school by bus, by car or on foot with the probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25. (9709/52/O/N/22 number 1) The probability that, on a randomly chosen day, Kino is late is 0.235.

   (a) Find the value of x.

   Let's draw a tree diagram with the given information,



   We are given that the probability that Kino is late on a randomly chosen day is,

$$P(L) = 0.235$$

   Let's write an expression for Kino being Late using our tree diagram,

$$P(L) = P(BL) + P(CL) + P(FL)$$

$$P(L) = (0.2 \times x) + (0.1 \times 2x) + (0.7 \times 0.25)$$

$$P(L) = 0.2x + 0.2x + 0.175$$

$$P(L) = 0.4x + 0.175$$

   Equate this to 0.235,

$$0.4x + 0.175 = 0.235$$

   Solve for x,

$$0.4x = 0.235 - 0.175$$

$$0.4x = 0.06$$

$$x = 0.15$$

   Therefore, the final answer is,

$$x = 0.15$$

   (b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late.

This is conditional probability,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

A - Kino is not late

B - Kino travels by car

A ∩ B - Kino is not late and travels by car

In the stem of the question, we are told that the probability that Kino is late on a randomly chosen day is 0.235. We can use that to find the probability that Kino is not late on a randomly chosen day,

$$P(A) = 1 - 0.235$$

$$P(A) = 0.765$$

Now let's find the probability that Kino is not late and he travels by car, read off the tree diagram,

$$P(A \cap B) = 0.1 \times (1 - 2x)$$
$$P(A \cap B) = 0.1 \times (1 - 2(0.15))$$
$$P(A \cap B) = 0.07$$

Now let's substitute into the formula for conditional probability,

$$P(B|A) = \frac{0.07}{0.765}$$

This simplifies to give,

$$P(B|A) = \frac{14}{153}$$

Therefore, the final answer is,

$$P(B|A) = \frac{14}{153}$$

8. Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time. (9709/52/O/N/22 number 5ab) Events A and B are defined as follows. A: all three coins show the same result B: at least one of the biased coin shows a head

   (a) Show that $P(B) = \frac{7}{16}$.

   Let's write down all the possible scenarios,

$$H \ BH_1 \ BH_2$$

$$H \ BH_1 \ BT_2$$

$$H \ BT_1 \ BH_2$$

$$T \ BH_1 \ BH_2$$

$$T \ BH_1 \ BT_2$$

$$T \ BT_1 \ BH_2$$

Note: BH represents the biased coin showing a head, BT the biased coin showing a tail, H the unbiased coin showing a head and T the unbiased coin showing tail.

Let's evaluate the first scenario,

$$P\left(H\ BH_1\ BH_2\right) = 0.5 \times 0.25 \times 0.25$$

$$P\left(H\ BH_1\ BH_2\right) = \frac{1}{32}$$

Let's evaluate the second scenario,

$$P\left(H\ BH_1\ BT_2\right) = 0.5 \times 0.25 \times 0.75$$

$$P\left(H\ BH_1\ BT_2\right) = \frac{3}{32}$$

Let's evaluate the third scenario,

$$P\left(H\ BT_1\ BH_2\right) = 0.5 \times 0.75 \times 0.25$$

$$P\left(H\ BT_1\ BH_2\right) = \frac{3}{32}$$

Let's evaluate the fourth scenario,

$$P\left(T\ BH_1\ BH_2\right) = 0.5 \times 0.25 \times 0.25$$

$$P\left(T\ BH_1\ BH_2\right) = \frac{1}{32}$$

Let's evaluate the fifth scenario,

$$P\left(T\ BH_1\ BT_2\right) = 0.5 \times 0.25 \times 0.75$$

$$P\left(T\ BH_1\ BT_2\right) = \frac{3}{32}$$

Let's evaluate the final scenario,

$$P\left(T\ BT_1\ BH_2\right) = 0.5 \times 0.75 \times 0.25$$

$$P\left(T\ BT_1\ BH_2\right) = \frac{3}{32}$$

Let's add up the results from all 6 scenarios,

$$P(B) = \frac{1}{32} + \frac{3}{32} + \frac{3}{32} + \frac{1}{32} + \frac{3}{32} + \frac{3}{32}$$

$$P(B) = \frac{7}{16}$$

Therefore, the final answer is,

$$P(B) = \frac{7}{16}$$

(b) Find $P(A|B)$.

The formula for conditional probability is,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Let's find $P(A \cap B)$, all three coins show the same result and at least one of the biased coin shows a head,

$$P(H\ BH_1\ BH_2) = 0.5 \times 0.25 \times 0.25$$

$$P(H\ BH_1\ BH_2) = \frac{1}{32}$$

$$P(A \cap B) = \frac{1}{32}$$

From part (a), we know that,

$$P(B) = \frac{7}{16}$$

Now let's substitute into the formula for conditional probability,

$$P(A|B) = \frac{1}{32} \div \frac{7}{16}$$

This simplifies to give,

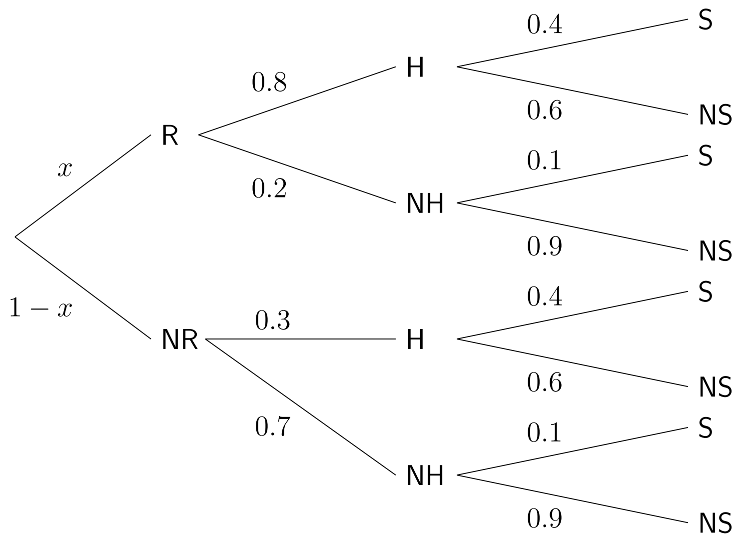
$$P(A|B) = \frac{1}{14}$$

Therefore, the final answer is,

$$P(A|B) = \frac{1}{14}$$

9. The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day if it is not raining and Aran is not wearing a hat or a scarf is 0.36. (9709/52/F/M/23 number 4) Find the value of x.

   Let's draw a tree diagram to represent the given information,



We are told that the probability that on a randomly chosen day if it is not raining and Aran is not wearing a hat or a scarf is 0.36,

$$P(NR \cap NH \cap NS) = 0.36$$

Let's find an expression for when it is not raining and Aran is not wearing a hart or a scarf, using our tree diagram,

$$P(NR \cap NH \cap NS) = (1 - x) \times 0.7 \times 0.9$$

$$P(NR \cap NH \cap NS) = (1 - x) \times 0.63$$

$$P(NR \cap NH \cap NS) = 0.63 - 0.63x$$

Let's equate this to 0.36,

$$0.63 - 0.63x = 0.36$$

Solve for x,

$$0.63x = 0.63 - 0.36$$

$$0.63x = 0.27$$

$$x = \frac{3}{7}$$

Therefore, the final answer is,

$$x = \frac{3}{7}$$