Probability and Statistics 1

5.4 Discrete Random Variables - Easy

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**AS Level**
Component:	**Probability and Statistics 1**
Topic:	**5.4 Discrete Random Variables**
Difficulty:	**Easy**

Questions

1. An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained. (9709/52/F/M/20 number 2)
   (a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6.
   On another occasion, this die is thrown 3 times. The random variable X is the number of times a 1 or a 6 is obtained.
   (b) Draw up the probability distribution table for X.
   (c) Find $E(X)$.
2. The score when the two fair six-sided die are thrown is the sum of the two numbers on the upper faces. (9709/51/M/J/20 number 1)
   (a) Show that the probability that the score is 4 is $\frac{1}{12}$.
   The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.
   (b) Find the mean of X.
   (c) Find the probability that a score of 4 is first obtained on the 6th throw.
   (d) Find $P(X < 8)$.
3. In a certain large college, 22% of the students own a car. (9709/53/M/J/20 number 2)
   (a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car.
   (b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4.
4. A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained. (9709/52/O/N/20 number 1)
   (a) Find the probability that obtaining a 4 requires fewer than 6 throws.
   On another occasion, the die is thrown 10 times.
   (b) Find the probability that a 4 is obtained at least 3 times.
5. A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes. (9709/52/O/N/20 number 2)
   (a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$.
   (b) Draw up the probability distribution table for X.
   (c) Given that $E(X) = \frac{15}{8}$, find $Var(X)$.
6. An ordinary fair die is thrown until a 6 is obtained. (9709/53/O/N/20 number 2)
   (a) Find the probability that obtaining a 6 takes more than 8 throws.
   Two ordinary far dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.
   (b) Find the expected value of X.
   (c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws.

7. A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands. (9709/52/F/M/21 number 1)

   (a) Find the probability that a score of 3 is obtained for the first time on the 8th spin.

   (b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time.

8. An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X. (9709/52/M/J/21 number 1)

   (a) Write down the mean of X.

   (b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw.

   (c) Find the probability that a 5 is first obtained in fewer than 10 throws.

9. Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time. (9709/51/O/N/21 number 1)

   (a) Find the probability that two tails are obtained for the first time on the 7th throw.

   (b) Find the probability that it takes more than 9 throws to obtain two tails for the first time.

10. A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement. (9709/52/O/N/21 number 3)

    (a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$.
    The random variable X is the number of yellow marbles selected.

    (b) Draw up the probability distribution table for X.

    (c) Find $E(X)$.

11. In a certain region, the probability that any given day in October is wet is 0.16, independently of other days. (9709/52/O/N/21 number 5)

    (a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet.

    (b) Find the probability that the first wet day in October is 8 October.

    (c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October.

12. A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered -3, -2, -1, -1. Each spinner is spun once and the number on the edge on which the spinner lands is noted. The random variable X denotes the sum of the resulting two numbers. (9709/52/F/M/22 number 1)

    (a) Draw up the probability distribution table for X.

    (b) Given that $E(X) = 0.25$, find the value of $Var(X)$.

13. A factory produces chocolates in three flavours: lemon, orange and strawberry in the ration 3:5:7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time. (9709/52/F/M/22 number 6ab)

    (a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks.

    (b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates.

14. A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained. (9709/52/M/J/22 number 2)

    (a) Draw up the probability distribution table for X.

    (b) Find $E(X)$ and $Var(X)$.

15. Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces. (9709/52/O/N/22 number 3)

    (a) Find the probability that a score of 17 or more is first obtained on the 6th throw.

    (b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws.

16. Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time. Events A and B are defined as follows.

    A : all three coins show the same result

    B : at least one of the biased coins shows a head

    The random variable X is the number of heads obtained when Eric throws the three coins. It is given that $P(B) = \frac{7}{16}$. (9709/52/O/N/22 number 5c) Draw up the probability distribution table for X.

17. Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which the spinner lands is recorded. The random variable X denotes the highest number recorded. (9709/53/O/N/22 number 4)

    (a) Show that $P(X = 2) = \frac{7}{64}$.

    (b) Complete the probability distribution table for X.

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

    On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

    (c) Find $P(Y = 6)$.

    (d) Find $P(Y > 4)$.

18. Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained. (9709/52/F/M/23 number 2)

    (a) Show that the probability of obtaining exactly one head is 0.225.

    (b) Complete the following probability distribution table for X.

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

    (c) Given that $E(X) = 2.1$, find the value of $Var(X)$.

19. Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X. (9709/53/M/J/23 number 1)

(a) State the value of $E(X)$.

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads.

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads.

20. (a) Hazeem repeatedly throws two ordinary fair 6-sided dice at the same time. On each occasion, the score is the sum of the two numbers that she obtains. (9709/51/O/N/23 number 2)

    i. Find the probability that it takes exactly 5 throws of the two dice for Hazeem to obtain a score of 8 or more.

    ii. Find the probability that it takes no more than 4 throws of the two dice for Hazeem to obtain a score of 8 or more.

    iii. For 8 randomly chosen throws of the dice, find the probability that Hazeem obtains a score of 8 or more on fewer than 3 occasions.

21. A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below. (9709/52/O/N/23 number 1)

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

(a) Given that $E(X) = 1.1$, find the value of p and the value of r.

(b) Find the numerical value of $Var(X)$.

22. A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same. (9709/52/M/J/20 number 5)

(a) Show that $P(X = 3) = \frac{7}{15}$.

(b) Draw up the probability distribution table for X.

(c) Find $E(X)$ and $Var(X)$.

Answers

1. An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained. (9709/52/F/M/20 number 2)

   (a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6.

   Let's start by defining our random variable,

   $$X\text{- r.v, number of throws until a } 1 \text{ or a } 6 \text{ is obtained}$$

   We know that this is a geometric distribution because we want the number of throws until a 1 or a 6 is obtained. Let's define our distribution,

   $$X \sim Geo(p)$$

   Our probability of success is the probability of obtaining a 1 or a 6,

   $$p = \frac{1}{6} + \frac{1}{6}$$
   $$p = \frac{1}{3}$$

   This means our distribution is,

   $$X \sim Geo\left(\frac{1}{3}\right)$$

   Now let's solve the question,

   $$P(3 \leq X \leq 5)$$

   We can rewrite that as,

   $$P(3 \leq X \leq 5) = P(X = 3, 4, 5)$$

   Let's evaluate those probabilities,

   $$P(3 \leq X \leq 5) = \left(\frac{2}{3}\right)^2 \times \frac{1}{3} + \left(\frac{2}{3}\right)^3 \times \frac{1}{3} + \left(\frac{2}{3}\right)^4 \times \frac{1}{3}$$

   This simplifies to give,

   $$P(3 \leq X \leq 5) = \frac{76}{243}$$

   Therefore, the final answer is,

   $$P(3 \leq X \leq 5) = \frac{76}{243}$$

   On another occasion, this die is thrown 3 times. The random variable X is the number of times a 1 or a 6 is obtained.

   (b) Draw up the probability distribution table for X.

Since the die is thrown 3 times, we can obtain either a 1 or a 6, 0, 1, 2 or 3 times,

$$X = 0, 1, 2, 3$$

Notice that the random variable X now follows a binomial distribution, since they have fixed the number of trials i.e the number of throws, and probability of success is constant,

$$X \sim B\left(3, \frac{1}{2}\right)$$

Let's find the probability that $X = 0$,

$$P(X = 0) = \left(\frac{2}{3}\right)^3$$

$$P(X = 0) = \frac{8}{27}$$

Let's find the probability that $X = 1$,

$$P(X = 1) = ^3C_1 \times \frac{1}{3} \times \left(\frac{2}{3}\right)^2$$

$$P(X = 1) = \frac{12}{27}$$

Let's find the probability that $X = 2$,

$$P(X = 2) = ^3C_2 \times \left(\frac{1}{3}\right)^2 \times \left(\frac{2}{3}\right)$$

$$P(X = 2) = \frac{6}{27}$$

Let's find the probability that $X = 3$,

$$P(X = 3) = \left(\frac{1}{3}\right)^3$$

$$P(X = 3) = \frac{1}{27}$$

Now let's draw the probability distribution table,

x	0	1	2	3
$P(X = x)$	$\frac{8}{27}$	$\frac{12}{27}$	$\frac{6}{27}$	$\frac{1}{27}$

(c) Find $E(X)$.

The formula for $E(X)$ is,

$$E(X) = \Sigma xp$$

Use the values from the probability distribution table,

$$E(X) = 0 \times \frac{8}{27} + 1 \times \frac{12}{27} + 2 \times \frac{6}{27} + 3 \times \frac{1}{27}$$

This simplifies to give,

$$E(X) = 1$$

Therefore, the final answer is,

$$E(X) = 1$$

2. The score when the two fair six-sided die are thrown is the sum of the two numbers on the upper faces. (9709/51/M/J/20 number 1)

   (a) Show that the probability that the score is 4 is $\frac{1}{12}$.

   Let's write down all the possible scenarios and evaluate them,

   $$\text{First die show a 1, and second die shows a 3} = \left(\frac{1}{6}\right)^2$$

   $$\text{First die show a 3, and second die shows a 1} = \left(\frac{1}{6}\right)^2$$

   $$\text{First die show a 2, and second die shows a 2} = \left(\frac{1}{6}\right)^2$$

   Let's add up the results of all three scenarios,

   $$\text{Total } = \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2$$

   $$\text{Total } = \frac{1}{12}$$

   Therefore, the final answer is,

   $$\frac{1}{12}$$

   The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

   (b) Find the mean of X.

   We know that this is a geometric distribution because of the word 'until'. The formula for mean under a geometric distribution is,

   $$E(X) = \frac{1}{p}$$

   In this case our probability of success is,

   $$p = \frac{1}{12}$$

Substitute into the formula for mean,

$$E(X) = 1 \div \frac{1}{12}$$

This simplifies to give,

$$E(X) = 12$$

Therefore, the final answer is,

$$E(X) = 12$$

(c) Find the probability that a score of 4 is first obtained on the 6th throw.

Let's define our distribution,

$$X \sim Geo\left(\frac{1}{12}\right)$$

The problem we are solving is,

$$P(X = 6)$$

This becomes,

$$P(X = 6) = \left(\frac{11}{12}\right)^5 \times \frac{1}{12}$$

Which simplifies to give,

$$P(X = 6) = 0.0539$$

(d) Find $P(X < 8)$.

We can rewrite this as,

$$P(X \leq 7)$$

Which allows us to use the formula,

$$P(X \leq r) = 1 - q^r$$

Use the formula above,

$$P(X \leq 7) = 1 - \left(\frac{11}{12}\right)^7$$

This simplifies to give,

$$P(X \leq 7) = 0.456$$

Therefore, the final answer is,

$$P(X < 8) = 0.456$$

3. In a certain large college, 22% of the students own a car. (9709/53/M/J/20 number 2)

   (a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car.

Let's define our random variable,

$$X - \text{r.v, number of students who own a car}$$

State the distribution of the random variable,

$$X \sim B(3, 0.22)$$

The problem we are solving is,

$$P(X = 3)$$

Using the formula for binomial probabilities,

$$P(X = 3) = 0.22^3$$

This simplifies to give,

$$0.0106$$

Therefore, the final answer is,

$$0.0106$$

(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4.

Let's define our random variable,

$$Y - \text{r.v, number of students who own a car}$$

State the distribution of the random variable,

$$Y \sim B(16, 0.22)$$

The problem we are solving is,

$$P(2 \leq Y \leq 4)$$

We can rewrite this as,

$$P(2 \leq Y \leq 4) = P(Y = 2, 3, 4)$$

Using the formula for binomial probabilities,

$$P(Y = 2, 3, 4) = {}^{16}C_2 \times 0.22^2 \times 0.78^{14} + {}^{16}C_3 \times 0.22^3 \times 0.78^{13} + {}^{16}C_4 \times 0.22^4 \times 0.78^{12}$$

This simplifies to give,

$$P(Y = 2, 3, 4) = 0.631$$

Therefore, the final answer is,

$$P(2 \leq Y \leq 4) = 0.631$$

4. A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained. (9709/52/O/N/20 number 1)

   (a) Find the probability that obtaining a 4 requires fewer than 6 throws.

Let's define our random variable,

$$X - \text{r.v, number of throws until a } 4 \text{ is obtained}$$

State the distribution of the random variable,

$$X \sim Geo\left(\frac{1}{6}\right)$$

The problem we are solving is,

$$P(X < 6)$$

We can rewrite this as,

$$P(X \leq 5)$$

Which allows us to use the formula,

$$P(X \leq r) = 1 - q^r$$

Using the formula,

$$P(X \leq 5) = 1 - \left(\frac{5}{6}\right)^5$$

This simplifies to give,

$$P(X \leq 5) = 0.598$$

Therefore, the final answer is,

$$P(X \leq 5) = 0.598$$

On another occasion, the die is thrown 10 times.

   (b) Find the probability that a 4 is obtained at least 3 times.

Notice that we now have a fixed number of throws, so our random variable now follows a Binomial distribution.

Let's define our random variable,

$$Y - \text{r.v, number of throws in which a } 4 \text{ is obtained}$$

State the distribution of the random variable,

$$Y \sim B\left(\frac{1}{6}\right)$$

The problem we are solving is,

$$P(Y \geq 3)$$

We can rewrite this as,

$$P(Y \geq 3) = 1 - P(Y < 3)$$

Note: We do this to simplify the calculation, otherwise you would have to calculate $P(X = 3, 4, 5, 6, 7, 8, 9, 10)$**.**

Using the formula for binomial probabilities,

$$P(Y \geq 3) = 1 - \left[{}^{10}C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^8 +^{10} C_1 \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^9 + \left(\frac{5}{6}\right)^{10}\right]$$

This simplifies to give,

$$P(Y \geq 3) = 1 - (0.290710 + 0.3230111 + 0.1615056)$$

$$P(Y \geq 3) = 0.225$$

Therefore, the final answer is,

$$P(Y \geq 3) = 0.225$$

5. A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes. (9709/52/O/N/20 number 2)

   (a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$.

   Let's assume that she picks a red ball followed by two blue balls,

   $$\underline{R}\ \underline{B}\ underline B$$

   The probability of this happening is,

   $$\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6}$$

   $$\frac{5}{56}$$

   Now we have to also consider that the order in which she picks these balls can be different. For example, she might pick the 2 **blue balls first and then the red ball,**

   $$\underline{B}\ \underline{B}\ \underline{R}$$

Let's find the number of total different arrangements,

$$\frac{3!}{2!}$$

$$3$$

Multiply the answer by 3**,**

$$\frac{5}{56} \times 3$$

This simplifies to give,

$$\frac{15}{56}$$

Therefore, the final answer is,

$$\frac{15}{56}$$

(b) Draw up the probability distribution table for X.

Sadie can either pick 0**,** 1**,** 2**,** 3 **red balls,**

$$X = 0, 1, 2, 3$$

Let's find the probability that $X = 0$**,**

$$P(X = 0) = P(BBB)$$

$$P(X = 0) = \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$$

$$P(X = 0) = \frac{1}{56}$$

We have already been given the probability that $X = 1$**,**

$$P(X = 1) = \frac{15}{56}$$

Let's find the probability that $X = 2$**,**

$$P(X = 2) = P(BRR) + P(RBR) + P(RRB)$$

$$P(X = 2) = \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} + \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

$$P(X = 2) = \frac{30}{56}$$

Let's find the probability that $X = 3$**,**

$$P(X = 3) = P(RRR)$$

$$P(X = 3) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$$

$$P(X = 3) = \frac{10}{56}$$

Now let's draw the probability distribution table for X,

x	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

(c) Given that $E(X) = \frac{15}{8}$, find $Var(X)$.

The formula for $Var(X)$ is,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

Let's find $\Sigma x^2 p$,

$$\Sigma x^2 p = 1^2 \times \frac{15}{56} + 2^2 \times \frac{30}{56} + 3^2 \times \frac{10}{56}$$
$$\Sigma x^2 p = \frac{225}{56}$$

Substitute into the formula for $Var(X)$,

$$Var(X) = \frac{225}{56} - \left[\frac{15}{8}\right]^2$$
$$Var(X) = \frac{225}{448}$$

Therefore, the final answer is,

$$Var(X) = \frac{225}{448}$$

6. An ordinary fair die is thrown until a 6 is obtained. (9709/53/O/N/20 number 2)

(a) Find the probability that obtaining a 6 takes more than 8 throws.

Let's define our random variable,

$X -$ r.v, number of throws until a 6 is obtained

State the distribution of the random variable,

$$X \sim Geo\left(\frac{1}{6}\right)$$

The problem we are solving is,

$$P(X > 8)$$

We can use the formula,

$$P(X > r) = q^r$$

Using the formula,

$$P(X > 8) = \left(\frac{5}{6}\right)^8$$

This simplifies to give,

$$P(X > 8) = 0.233$$

Therefore, the final answer is,

$$P(X > 8) = 0.233$$

Two ordinary far dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X.

The mean for a geometric distribution is,

$$E(X) = \frac{1}{p}$$

The probability of getting a pair of 6s is,

$$p = \frac{1}{6} \times \frac{1}{6}$$

$$p = \frac{1}{36}$$

Substitute into the formula,

$$E(X) = 1 \div \frac{1}{36}$$

Which simplifies to give,

$$E(X) = 36$$

Therefore, the final answer is,

$$E(X) = 36$$

(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws.

Let's state the distribution of our random variable, X,

$$X \sim Geo\left(\frac{1}{36}\right)$$

We are required to find,

$$P(X = 10, 11)$$

Using the formula for geometric probabilities,

$$P(X = 10, 11) = \left(\frac{1}{36}\right) \times \left(\frac{35}{36}\right)^{9} + \left(\frac{1}{36}\right) \times \left(\frac{35}{36}\right)^{10}$$

This simplifies to give,

$$P(X = 10, 11) = 0.0425$$

Therefore, the final answer is,

$$P(X = 10, 11) = 0.0425$$

7. A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands. (9709/52/F/M/21 number 1)

   (a) Find the probability that a score of 3 is obtained for the first time on the 8th spin.

   Let's define our random variable,

   $$X - \text{r.v, number of throws it takes to obtain a 3 for the first time}$$

   State the distribution of the random variable,

   $$X \sim Geo\left(\frac{1}{5}\right)$$

   The problem we are solving is,

   $$P(X = 8)$$

   Using the formula for geometric probabilities,

   $$P(X = 8) = \left(\frac{4}{5}\right)^7 \times \frac{1}{5}$$

   This simplifies to give,

   $$P(X = 8) = 0.0419$$

   Therefore, the final answer is,

   $$P(X = 8) = 0.0419$$

   (b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time.

   We are solving,

   $$P(X < 6)$$

   We can rewrite this as,

   $$P(X \leq 5)$$

   Which allows us to use this formula,

   $$P(X \leq r) = 1 - q^r$$

   Using the formula,

   $$P(X \leq 5) = 1 - \left(\frac{4}{5}\right)^5$$

Which simplifies to give,

$$P(X \leq 5) = 0.672$$

Therefore, the final answer is,

$$P(X \leq 5) = 0.672$$

8. An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X. (9709/52/M/J/21 number 1)

   (a) Write down the mean of X.

   The formula for mean of a geometric distribution is,

   $$E(X) = \frac{1}{p}$$

   The probability of getting a 5 is,

   $$p = \frac{1}{6}$$

   Substitute into the formula,

   $$E(X) = 1 \div \frac{1}{6}$$

   Which simplifies to give,

   $$E(X) = 6$$

   Therefore, the final answer is,

   $$E(X) = 6$$

   (b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw.

   Let's define our random variable,

   $$X - \text{r.v, number of throws it takes to obtain a } 5 \text{ for the first time}$$

   State the distribution of the random variable,

   $$X \sim Geo\left(\frac{1}{6}\right)$$

   The problem we are solving is,

   $$P(3 < X < 8)$$

   We can rewrite this as,

   $$P(3 < X < 8) = P(X = 4, 5, 6, 7)$$

   Using the formula for geometric probabilities,

   $$P(X = 4, 5, 6, 7) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6}$$

This simplifies to give,

$$P(X = 4, 5, 6, 7) = 0.300$$

Therefore, the final answer is,

$$P(X = 4, 5, 6, 7) = 0.300$$

(c) Find the probability that a 5 is first obtained in fewer than 10 throws.

We are solving,

$$P(X < 10)$$

We can rewrite this as,

$$P(X \leq 9)$$

So that we can use the formula,

$$P(X \leq r) = 1 - q^r$$

Using the formula,

$$P(X \leq 9) = 1 - \left(\frac{5}{6}\right)^9$$

This simplifies to give,

$$P(X \leq 9) = 0.806$$

Therefore, the final answer is,

$$P(X \leq 9) = 0.806$$

9. Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time. (9709/51/O/N/21 number 1)

(a) Find the probability that two tails are obtained for the first time on the 7th throw.

State the distribution of X,

$$X \sim Geo(p)$$

The probability required to obtain two tails is,

$$p = 0.5 \times 0.5$$

$$p = 0.25$$

This means that our distribution is,

$$X \sim Geo(0.25)$$

The problem we are required to solve is,

$$P(X = 7)$$

Using the formula for geometric probabilities,

$$P(X = 7) = 0.75^6 \times 0.25$$

This simplifies to give,

$$P(X = 7) = 0.0445$$

Therefore, the final answer is,

$$P(X = 7) = 0.0445$$

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time.

We are solving,

$$P(X > 9)$$

We can use the formula,

$$P(X > r) = q^r$$

Using the formula,

$$P(X > 9) = 0.75^9$$

This simplifies to give,

$$P(X > 9) = 0.0751$$

Therefore, the final answer is,

$$P(X > 9) = 0.0751$$

10. A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement. (9709/52/O/N/21 number 3)

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$.

Let's assume that the three marbles are selected in the order yellow, green, green,

$$\underline{Y}\ \underline{G}\ \underline{G}$$

The probabilities would be,

$$\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7}$$
$$\frac{5}{42}$$

However, the order can also change. Let's find the number of different arrangements of yellow, green, green,

$$\frac{3!}{2!}$$
$$3$$

So we multiply our answer by 3,

$$\frac{5}{42} \times 3$$

This simplifies to give,

$$\frac{15}{42}$$

Which can be further simplified to,

$$\frac{5}{14}$$

Therefore, the final answer is,

$$\frac{5}{14}$$

The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X.

We can select 0, 1, 2 or 3 yellow marbles,

$$X = 0, 1, 2, 3$$

Let's find the probability that $X = 0$,

$$P(X = 0) = P(GGG)$$
$$P(X = 0) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}$$
$$P(X = 0) = \frac{2}{42}$$

We already evaluated $P(X = 1)$ in part (a),

$$P(X = 1) = \frac{15}{42}$$

Let's find the probability that $X = 2$,

$$P(X = 2) = P(YYG) \times \frac{3!}{2!}$$
$$P(X = 2) = \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times \frac{3!}{2!}$$
$$P(X = 2) = \frac{20}{42}$$

Let's find the probability that $X = 3$,

$$P(X = 3) = P(YYY)$$

$$P(X = 3) = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7}$$

$$P(X = 3) = \frac{5}{42}$$

Now let's draw the probability distribution table,

x	0	1	2	3
$P(X = x)$	$\frac{2}{42}$	$\frac{15}{42}$	$\frac{20}{42}$	$\frac{5}{42}$

(c) Find $E(X)$.

The formula for mean is,

$$E(X) = \Sigma xp$$

Use the values in the probability distribution table,

$$E(X) = 0 \times \frac{2}{42} + 1 \times \frac{15}{42} + 2 \times \frac{20}{42} + 3 \times \frac{5}{42}$$

This simplifies to give,

$$E(X) = \frac{5}{3}$$

Therefore, the final answer is,

$$E(X) = \frac{5}{3}$$

11. In a certain region, the probability that any given day in October is wet is 0.16, independently of other days. (9709/52/O/N/21 number 5)

(a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet.

Let's define our random variable,

$$X - \text{r.v, number of days that are wet in October}$$

Define the distribution,

$$X \sim B(10, 0.16)$$

We are solving,

$$P(X < 3)$$

Using the formula for binomial probabilities,

$$P(X < 3) =^{10} C_2 \times 0.16^2 \times 0.84^8 +^{10} C_1 \times 0.16 \times 0.84^9 + 0.84^{10}$$

This simplifies to give,

$$P(X < 3) = 0.794$$

Therefore, the final answer is,

$$P(X < 3) = 0.794$$

(b) Find the probability that the first wet day in October is 8 October.

Let's define our random variable,

$Y -$ r.v, number of days until the first wet day in October

Define the distribution,

$$Y \sim Geo0.16$$

We are solving,

$$P(Y = 8)$$

Using the formula for geometric probabilities,

$$P(Y = 8) = 0.84^7 \times 0.16$$

This simplifies to give,

$$P(Y = 8) = 0.0472$$

Therefore, the final answer is,

$$P(Y = 8) = 0.0472$$

(c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October.

Let's define our random variable,

$V -$ r.v, number of years in which the first wet day in Ocober is 8 October

Define the distribution,

$$V \sim B(4, 0.0472)$$

Note: p is our answer for part (b).

We are solving,

$$P(V = 1)$$

Using the formula for binomial probabilities,

$$P(V = 1) =^4 C_1 \times 0.0472 \times (1 - 0.0472)^3$$

This simplifies to give,

$$P(V = 1) = 0.163$$

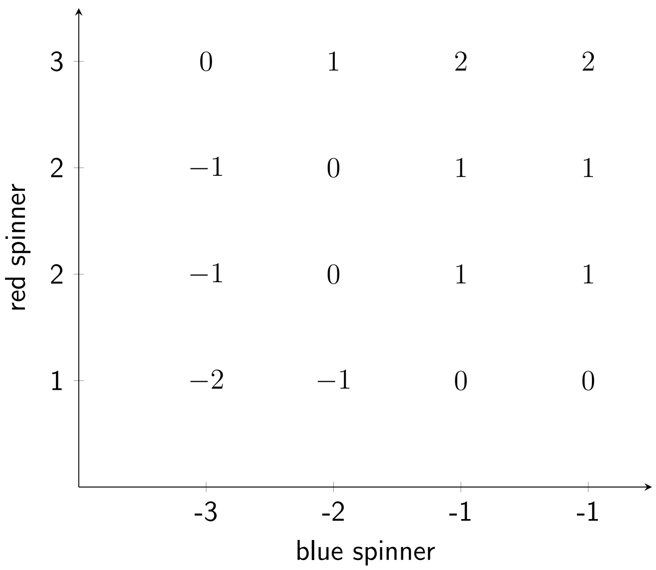
Therefore, the final answer is,

$$P(V = 1) = 0.163$$

12. A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered -3, -2, -1, -1. Each spinner is spun once and the number on the edge on which the spinner lands is noted. The random variable X denotes the sum of the resulting two numbers. (9709/52/F/M/22 number 1)

   (a) Draw up the probability distribution table for X.

   Let's start by drawing a possibility space diagram,



   We can read off our probabilities from the diagram. Now let's draw our probability distribution table,

x	-2	-1	0	1	2
$P(X = x)$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{2}{16}$

   (b) Given that $E(X) = 0.25$, find the value of $Var(X)$.

   The formula for variance is,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

   Let's find $\Sigma x^2 p$ using the values from the probability distribution table,

$$\Sigma x^2 p = (-2)^2 \times \frac{1}{16} + (-1)^2 \times \frac{3}{16} + 0^2 \times \frac{5}{16} + 1^2 \times \frac{5}{16} + 2^2 \times \frac{2}{16}$$

$$\Sigma x^2 p = \frac{5}{4}$$

Substitute into the formula for variance,

$$Var(X) = \frac{5}{4} - (0.25)^2$$

This simplifies to give,

$$Var(X) = \frac{19}{16}$$

Therefore, the final answer is,

$$Var(X) = \frac{19}{16}$$

13. A factory produces chocolates in three flavours: lemon, orange and strawberry in the ration 3:5:7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time. (9709/52/F/M/22 number 6ab)

    (a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks.

Let's define our random variable,

$X -$ r.v, number of chocolates checked until Nell finds a lemon flavoured chocolate

Define the distribution,

$$X \sim Geo(p)$$

The probability of getting a lemon flavoured chocolate is,

$$p = \frac{3}{15}$$

$$p = \frac{1}{5}$$

This means our distribution is,

$$X \sim Geo\left(\frac{1}{5}\right)$$

We are solving,

$$P(X = 7)$$

Using the formula for geometric probabilities,

$$P(X = 7) = \left(\frac{4}{5}\right)^6 \times \frac{1}{5}$$

This simplifies to give,

$$P(X = 7) = 0.0524$$

Therefore, the final answer is,

$$P(X = 7) = 0.0524$$

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates.

We are solving,

$$P(X \geq 6)$$

We can rewrite this as,

$$P(X > 5)$$

This allows us to use the formula,

$$P(X > r) = q^r$$

Using the formula,

$$P(X > 5) = \left(\frac{4}{5}\right)^5$$

This simplifies to give,

$$P(X > 5) = 0.328$$

Therefore, the final answer is,

$$P(X > 5) = 0.328$$

14. A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained. (9709/52/M/J/22 number 2)

(a) Draw up the probability distribution table for X.

Let's start by drawing a possibility space diagram,

roll 1 \ roll 2	1	2	2	3	3	3
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6
2	3	4	4	5	5	5
2	3	4	4	5	5	5
1	2	3	3	4	4	4

Read off the values from the diagram. Now let's draw the probability distribution table,

x	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

(b) Find $E(X)$ and $Var(X)$.

Let's start by finding the mean. The formula for mean is,

$$E(X) = \Sigma xp$$

Read off the values from the probability distribution table,

$$E(X) = 2 \times \frac{1}{36} + 3 \times \frac{4}{36} + 4 \times \frac{10}{36} + 5 \times \frac{12}{36} + 6 \times \frac{9}{36}$$

This simplifies to give,

$$E(X) = \frac{14}{3}$$

Now let's find the variance. The formula for variance is,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

Let's find $\Sigma x^2 p$,

$$\Sigma x^2 p = 2^2 \times \frac{1}{36} + 3^2 \times \frac{4}{36} + 4^2 \times \frac{10}{36} + 5^2 \times \frac{12}{36} + 6^2 \times \frac{9}{36}$$

$$\Sigma x^2 p = \frac{206}{9}$$

Substitute into the formula for variance,

$$Var(X) = \frac{206}{9} - \left(\frac{14}{3}\right)^2$$

This simplifies to give,

$$Var(X) = \frac{10}{9}$$

Therefore, the final answer is,

$$E(X) = \frac{14}{3} \qquad Var(X) = \frac{10}{9}$$

15. Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces. (9709/52/O/N/22 number 3)

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw.

Let's evaluate all the possible scenarios for obtaining a score of 17 or more,

$$\text{First die shows a 5, second die shows a 6, third die shows a 6} = \left(\frac{1}{6}\right)^3$$

$$\text{First die shows a 6, second die shows a 5, third die shows a 6} = \left(\frac{1}{6}\right)^3$$

$$\text{First die shows a 6, second die shows a 6, third die shows a 5} = \left(\frac{1}{6}\right)^3$$

$$\text{First die shows a 6, second die shows a 6, third die shows a 6} = \left(\frac{1}{6}\right)^3$$

Add up the results of all four scenarios,

$$\text{Total } = \left(\frac{1}{6}\right)^3 \times 4$$

$$\text{Total } = \frac{4}{216}$$

Let's define our random variable,

$$Y - \text{r.v, number of throws until a score of } 17 \text{ or more is first obtained}$$

Define the distribution,

$$Y \sim Geo\left(\frac{4}{216}\right)$$

We are solving,

$$P(Y = 6)$$

Using the formula for geometric probabilities,

$$P(Y = 6) = \left(\frac{212}{216}\right)^5 \times \frac{4}{216}$$

This simplifies to give,

$$P(Y = 6) = 0.0169$$

Therefore, the final answer is,

$$P(Y = 6) = 0.0169$$

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws.

We are solving,

$$P(Y < 8)$$

We can rewrite this as,

$$P(Y \leq 7)$$

Which allows us to use the formula,

$$P(Y \leq r) = 1 - q^r$$

Using the formula,

$$P(Y \leq 7) = 1 - \left(\frac{212}{216}\right)^7$$

This simplifies to give,

$$P(Y \leq 7) = 0.123$$

Therefore, the final answer is,

$$P(Y \leq 7) = 0.123$$

16. Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time. Events A and B are defined as follows.

A : all three coins show the same result

B : at least one of the biased coins shows a head

The random variable X is the number of heads obtained when Eric throws the three coins. It is given that $P(B) = \frac{7}{16}$. (9709/52/O/N/22 number 5c) Draw up the probability distribution table for X.

X **can take the following values,**

$$X = 0, 1, 2, 3$$

Let's find the probability that $X = 0$,

$$P(X = 0) = P\left(T\ BT_1\ BT_2\right)$$

$$P(X = 0) = 0.5 \times 0.75 \times 0.75$$

$$P(X = 0) = \frac{9}{32}$$

Let's find the probability that $X = 1$,

$$P(X = 1) = P\left(H\ BT_1\ BT_2\right) + P\left(T\ BH_1\ BT_2\right) + P\left(T\ BT_1\ BH_2\right)$$

$$P(X = 1) = 0.5 \times 0.75 \times 0.75 + 0.5 \times 0.25 \times 0.75 + 0.5 \times 0.75 \times 0.25$$

$$P(X = 1) = \frac{15}{32}$$

Let's find the probability that $X = 2$,

$$P(X = 2) = P\left(H\ BH_1\ BT_2\right) + P\left(T\ BH_1\ BH_2\right) + P\left(H\ BT_1\ BH_2\right)$$

$$P(X = 2) = 0.5 \times 0.25 \times 0.75 + 0.5 \times 0.25 \times 0.25 + 0.5 \times 0.75 \times 0.25$$

$$P(X = 2) = \frac{7}{32}$$

Let's find the probability that $X = 3$,

$$P(X = 3) = P(H\ BH_1\ BH_2)$$

$$P(X = 3) = 0.5 \times 0.25 \times 0.25$$

$$P(X = 3) = \frac{1}{32}$$

Now let's draw the probability distribution table,

x	0	1	2	3
$P(X = x)$	$\frac{9}{32}$	$\frac{15}{32}$	$\frac{7}{32}$	$\frac{1}{32}$

17. Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which the spinner lands is recorded. The random variable X denotes the highest number recorded. (9709/53/O/N/22 number 4)

(a) Show that $P(X = 2) = \frac{7}{64}$.

Let's evaluate all the possible scenarios,

All spinners show a 2 $= 0.25^3$

One spinner shows a 1 and the others show a 2 $= 0.25^3 \times 3$

One spinner shows a 2 and the others show a 1 $= 0.25^3 \times 3$

Note: We multiply by 3 because it can be any of the three spinners that show a 2, while the others show a 1. This is the same logic that is applied in the second scenario.

Let's add up the results of all three scenarios,

$$\text{Total } = 0.25^3 + 0.25^3 \times 3 + 0.25^3 \times 3$$

$$\text{Total } = \frac{7}{64}$$

Therefore, the final answer is,

$$P(X = 2) = \frac{7}{64}$$

(b) Complete the probability distribution table for X.

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

Let's find the probability that $X = 1$. This is only possible if all three spinners show a 1,

$$P(X = 1) = 0.25^3$$

$$P(X = 1) = \frac{1}{64}$$

Now let's find the probability that $X = 4$, to do this we can subtract all the probabilities from 1,

$$P(X = 4) = 1 - \frac{1}{64} - \frac{7}{64} - \frac{19}{64}$$
$$P(X = 4) = \frac{37}{64}$$

Note: Solving it by evaluating all the possible scenarios would be tedious hence the use of $P(X = x) = 1$.

Now let's fill in the probability distribution table,

x	1	2	3	4
$P(X = x)$	$\frac{1}{64}$	$\frac{7}{64}$	$\frac{19}{64}$	$\frac{37}{64}$

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$.

Let's define the distribution for Y,

$$Y \sim Geo\left(\frac{1}{4}\right)$$

Use the formula for geometric probabilities,

$$P(Y = 6) = \left(\frac{3}{4}\right)^5 \times \frac{1}{4}$$

This simplifies to give,

$$P(Y = 6) = 0.0593$$

Therefore, the final answer i,s

$$P(Y = 6) = 0.0593$$

(d) Find $P(Y > 4)$.

We can use the formula,

$$P(Y > r) = q^r$$

Using the formula,

$$P(Y > 4) = \left(\frac{3}{4}\right)^4$$

This simplifies to give,

$$P(Y > 4) = 0.316$$

Therefore, the final answer is,

$$P(Y > 4) = 0.316$$

18. Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained. (9709/52/F/M/23 number 2)

(a) Show that the probability of obtaining exactly one head is 0.225.

We can write this as,

$$P(BH\ T\ T\ T) + 3 \times P(BT\ H\ T\ T)$$

$$0.6 \times 0.5^3 + 3\left(0.4 \times 0.5^3\right)$$

$$0.225$$

Therefore, the final answer is,

$$0.225$$

(b) Complete the following probability distribution table for X.

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

Let's find the probability that $X = 3$,

$$P(X = 3) = 3 \times P(BH\ H\ H\ T) + P(BT\ H\ H\ H)$$

$$P(X = 3) = 3\left(0.6 \times 0.5^3\right) + 0.4 \times 0.5^3$$

$$P(X = 3) = 0.275$$

We can get $X = 2$ by subtracting all the probabilities from 1,

$$P(X = 2) = 1 - (0.05 + 0.225 + 0.275 + 0.075)$$

$$P(X = 2) = 0.375$$

Now let's fill in the probability distribution table,

x	0	1	2	3	4
$P(X = x)$	0.05	0.225	0.375	0.275	0.075

(c) Given that $E(X) = 2.1$, find the value of $Var(X)$.

The formula for variance is,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

Let's find $\Sigma x^2 p$,

$$\Sigma x^2 p = 1^2 \times 0.225 + 2^2 \times 0.375 + 3^2 \times 0.275 + 4^2 \times 0.075$$

$$\Sigma x^2 p = 5.4$$

Substitute into the formula for variance,

$$Var(X) = 5.4 - (2.1)^2$$

$$Var(X) = 0.99$$

Therefore, the final answer is,

$$Var(X) = 0.99$$

19. Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X. (9709/53/M/J/23 number 1)

(a) State the value of $E(X)$.

The formula for mean under a geometric distribution is,

$$E(X) = \frac{1}{p}$$

The probability of obtaining a pair of heads is,

$$p = 0.5 \times 0.5$$

$$p0.25$$

Substitute into the formula,

$$E(X) = \frac{1}{0.25}$$

This simplifies to give,

$$E(X) = 4$$

Therefore, the final answer is,

$$E(X) = 4$$

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads.

Define the distribution for X,

$$X \sim Geo(0.25)$$

We are solving,

$$P(X = 5)$$

Use the formula for geometric probabilities,

$$P(X = 5) = (0.75)^4 \times 0.25$$

This simplifies to give,

$$P(X = 5) = 0.0791$$

Therefore, the final answer is,

$$P(X = 5) = 0.0791$$

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads.

We are solving,

$$P(X < 7)$$

We can rewrite this as,

$$P(X \leq 6)$$

Which allows us to use the formula,

$$P(X \leq r) = 1 - q^r$$

Using the formula,

$$P(X \leq 6) = 1 - 0.75^6$$

This simplifies to give,

$$P(X \leq 6) = 0.822$$

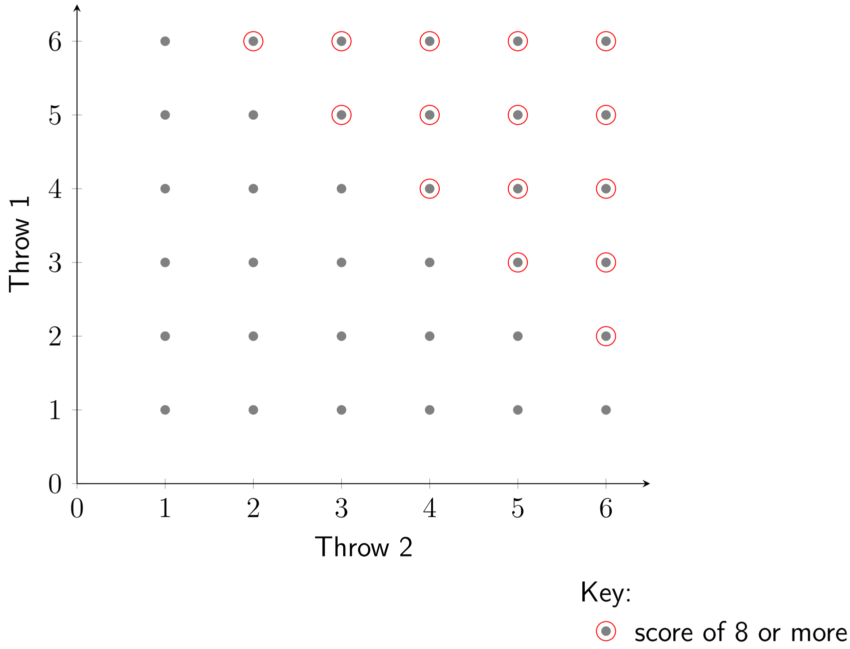
Therefore, the final answer is,

$$P(X \leq 6) = 0.822$$

20. (a) Hazeem repeatedly throws two ordinary fair 6-sided dice at the same time. On each occasion, the score is the sum of the two numbers that she obtains. (9709/51/O/N/23 number 2)

i. Find the probability that it takes exactly 5 throws of the two dice for Hazeem to obtain a score of 8 or more.

Let's start by drawing a possibility space diagram,



Let's define our random variable,

$$X - \text{r.v, number of throws until a score of } 8 \text{ or more is obtained}$$

Let's define its distribution,

$$X \sim Geo\left(\frac{15}{36}\right)$$

Note: We deduced the value of p from the possibility space diagram.

We are solving,

$$P(X = 5)$$

Use the formula for geometric probabilities,

$$P(X = 5) = \left(\frac{21}{36}\right)^4 \times \frac{15}{36}$$

This simplifies to give,

$$P(X = 5) = 0.0482$$

Therefore, the final answer is,

$$P(X = 5) = 0.0482$$

ii. Find the probability that it takes no more than 4 throws of the two dice for Hazeem to obtain a score of 8 or more.

We are solving,

$$P(X \leq 4)$$

Which allows us to use the formula,

$$P(X \leq r) = 1 - q^r$$

Using the formula,

$$P(X \leq 4) = 1 - \left(\frac{21}{36}\right)^4$$

This simplifies to give,

$$P(X \leq 4) = 0.884$$

Therefore, the final answer is,

$$P(X \leq 4) = 0.884$$

iii. For 8 randomly chosen throws of the dice, find the probability that Hazeem obtains a score of 8 or more on fewer than 3 occasions.

Let's define our random variable,

Y – r.v, number of throws on which the score is 8 or more

Let's define our distribution,

$$Y \sim B\left(8, \frac{15}{36}\right)$$

We are solving,

$$P(Y < 3)$$

Use the formula for binomial probabilities,

$$P(Y < 3) = \left(\frac{21}{36}\right)^8 + {}^8C_1 \times \frac{15}{36} \times \left(\frac{21}{36}\right)^7 + {}^8C_2 \times \left(\frac{15}{36}\right)^2 \times \left(\frac{21}{36}\right)^6$$

This simplifies to give,

$$P(Y < 3) = 0.282$$

Therefore, the final answer is,

$$P(Y < 3) = 0.282$$

21. A competitor in a throwing event has three attempts to throw a ball as far as possible. The random variable X denotes the number of throws that exceed 30 metres. The probability distribution table for X is shown below. (9709/52/O/N/23 number 1)

x	0	1	2	3
$P(X = x)$	0.4	p	r	0.15

(a) Given that $E(X) = 1.1$, find the value of p and the value of r.

Let's create an expression for $E(X)$,

$$E(X) = 0 \times 0.4 + 1 \times p + 2 \times r + 3 \times 0.15$$

$$E(X) = p + 2r + 0.45$$

Equate this to 1.1 and simplify,

$$p + 2r + 0.45 = 1.1$$

$$p = 0.65 - 2r$$

All the probabilities, $P(X = x)$, sum up to 1,

$$P(X = x) = 1$$

$$0.4 + p + r + 0.15 = 1$$
$$p + r + 0.55 = 1$$
$$p + r = 0.45$$

We now have two equations that we can solve simultaneously,

$$p = 0.65 - 2r \quad p + r = 0.45$$

Substitute the second equation into the first equation,

$$p + r = 0.45$$

$$0.65 - 2r + r = 0.45$$
$$0.65 - r = 0.45$$
$$r = 0.2$$

Now let's find p,

$$p = 0.65 - 2r$$
$$p = 0.65 - 2(0.2)$$
$$p = 0.25$$

Therefore, the final answer is,

$$p = 0.25 \quad r = 0.2$$

(b) Find the numerical value of $Var(X)$.

The formula for variance is,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

Let's find $\Sigma x^2 p$,

$$\Sigma x^2 p = 1^2 \times 0.25 + 2^2 \times 0.2 + 3^2 \times 0.15$$
$$\Sigma x^2 p = 2.4$$

Substitute into the formula for variance,

$$Var(X) = 2.4 - (1.1)^2$$

$$Var(X) = 1.19$$

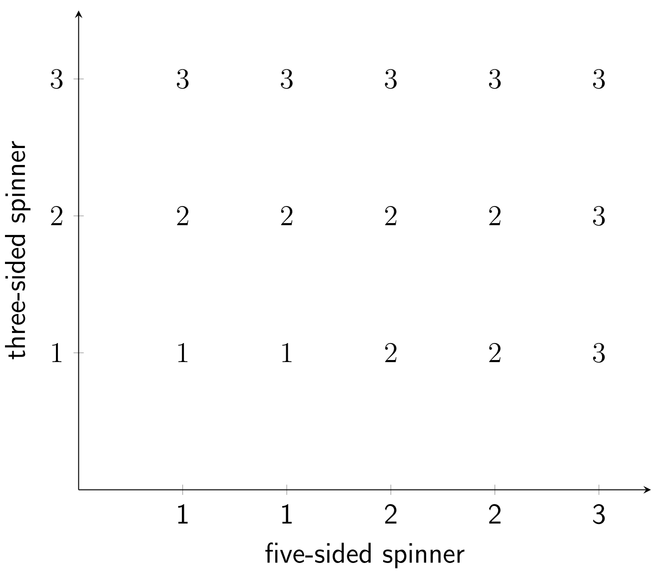
Therefore, the final answer is,

$$Var(X) = 1.19$$

22. A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same. (9709/52/M/J/20 number 5)

 (a) Show that $P(X = 3) = \frac{7}{15}$.

 Let's start by drawing a possibility space diagram,



 Read off the value from the diagram,

$$P(X = 3) = \frac{7}{15}$$

 Therefore, the final answer is,

$$P(X = 3) = \frac{7}{15}$$

 (b) Draw up the probability distribution table for X.

Read off the values from the possibility space diagram,

x	1	2	3
$P(X = x)$	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{7}{15}$

(c) Find $E(X)$ and $Var(X)$.

Let's start by finding the mean,

$$E(X) = \Sigma xp$$

Use the values in the probability distribution table,

$$E(X) = 1 \times \frac{2}{15} + 2 \times \frac{6}{15} + 3 \times \frac{7}{15}$$

This simplifies to give,

$$E(X) = \frac{7}{3}$$

Now let's find the variance,

$$Var(X) = \Sigma x^2 p - [E(X)]^2$$

Let's find $\Sigma x^2 p$,

$$\Sigma x^2 p = 1^2 \times \frac{2}{15} + 2^2 \times \frac{6}{15} + 3^2 \times \frac{7}{15}$$

$$\Sigma x^2 p = \frac{89}{15}$$

Substitute into the formula for variance,

$$Var(X) = \frac{89}{15} - \left(\frac{7}{3}\right)^2$$

$$Var(X) = \frac{22}{45}$$

Therefore, the final answer is,

$$E(X) = \frac{7}{15} \quad Var(X) = \frac{22}{45}$$