

Pure Maths 3

3.1 Algebra - Easy



Subject:	Mathematics
Syllabus Code:	9709
Level:	A2 Level
Component:	Pure Mathematics 3
Topic:	3.1 Algebra
Difficulty:	Easy

Questions

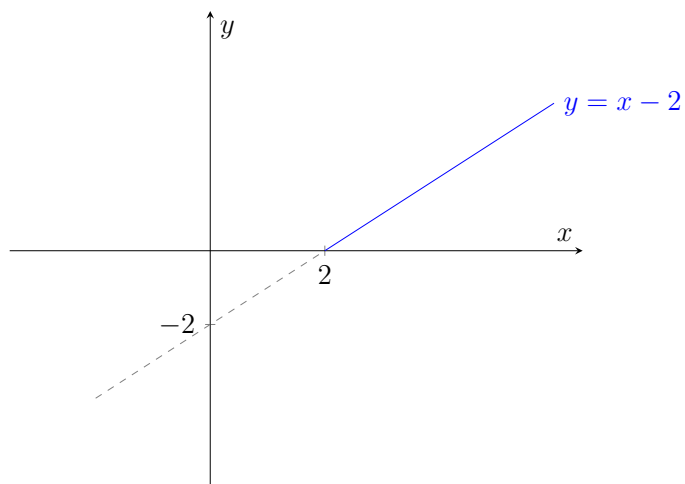
- Sketch the graph of $y = |x - 2|$. (9709/32/F/M/20 number 1)
 - Solve the inequality $|x - 2| < 3x - 4$.
- Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. (9709/32/M/J/20 number 1)
- Solve the inequality $|2x - 1| > 3|x + 2|$. (9709/33/M/J/20 number 1)
- Solve the inequality $2 - 5x > 2|x - 3|$. (9709/31/O/N/20 number 1)
- Solve the inequality $2|3x - 1| < |x + 1|$. (9709/31/M/J/21 number 1)
- Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. (9709/32/O/N/21 number 2)
- Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. (9709/33/O/N/21 number 1)
- Solve the inequality $|2x + 3| > 3|x + 2|$. (9709/32/F/M/22 number 1)
- Sketch the graph of $y = |2x + 1|$. (9709/31/O/N/22 number 1)
 - Solve the inequality $3x + 5 < |2x + 1|$.
- Sketch the graph of $y = |2x + 3|$. (9709/31/M/J/23 number 2)
 - Solve the inequality $3x + 8 > |2x + 3|$.
- The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$. Show that $(x + 3)$ is a factor of $p(x)$. (9709/31/M/J/23 number 10a)
- Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. (9709/33/M/J/23 number 2)

Answers

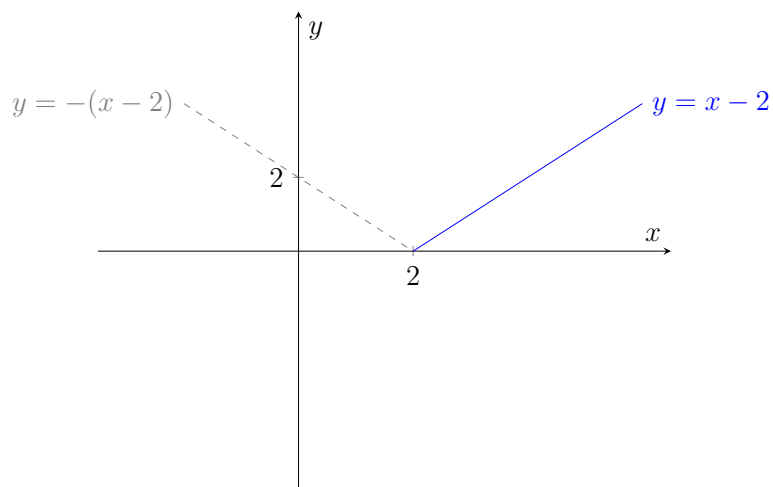
1. (a) Sketch the graph of $y = |x - 2|$. (9709/32/F/M/20 number 1)

$$y = |x - 2|$$

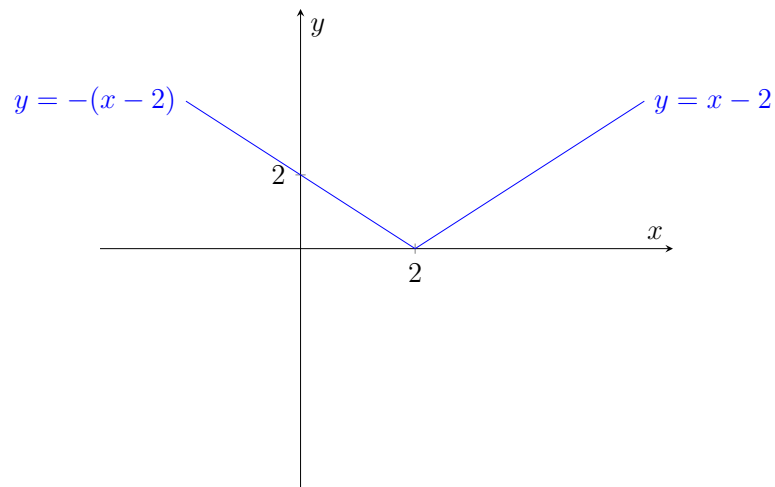
Start by sketching the graph of $y = x - 2$,



Reflect the part of the graph that's below the x -axis,



Therefore, the final answer is,

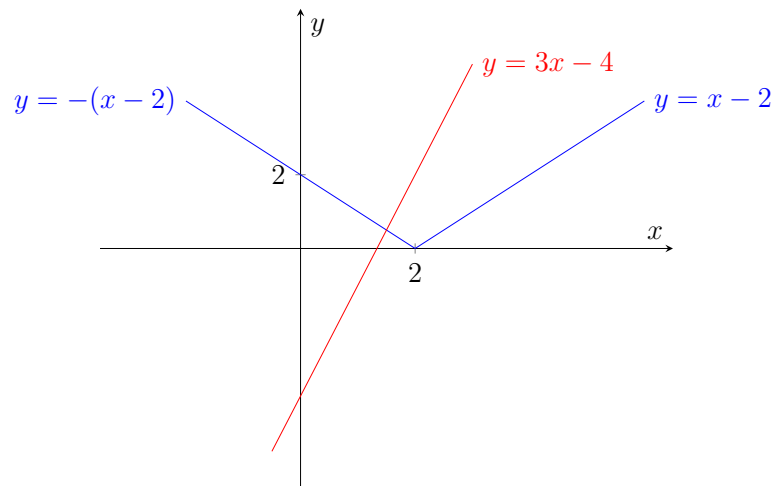


Note: Ensure that you label both lines.

(b) Solve the inequality $|x - 2| < 3x - 4$.

$$|x - 2| < 3x - 4$$

Let's start by sketching the graph of $y = 3x - 4$ on the same plane as the graph of $y = |x - 2|$,



Identify the which lines are intersecting and solve their equations simultaneously,

$$y = -(x - 2) \quad y = 3x - 4$$

$$-(x - 2) = 3x - 4$$

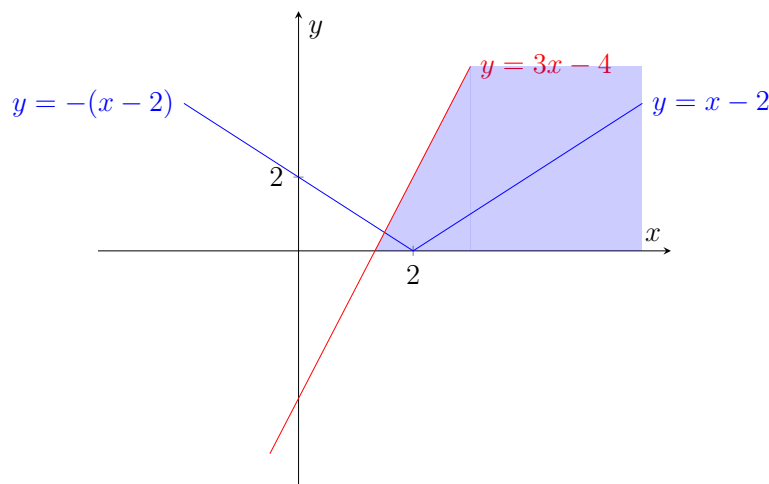
$$-x + 2 = 3x - 4$$

$$4x - 6 = 0$$

$$4x = 6$$

$$x = \frac{3}{2}$$

Now we know that $y = 3x - 4$ and $y = |x - 2|$ intersect when $x = \frac{3}{2}$. Let's identify the region that satisfies our inequality,



Therefore, the final answer is,

$$x > \frac{3}{2}$$

2. Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. (9709/32/M/J/20 number 1)

$$2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6}$$

Divide $6x^4$ by $2x^2$,

$$2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \quad 3x^2$$

Multiply $3x^2$ by the divisor, $2x^2 - x + 1$,

$$2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \quad 3x^2$$

$$\underline{-(6x^4 - 3x^3 + 3x^2)} $$

Subtract,

$$2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \quad 3x^2$$

$$\underline{-(6x^4 - 3x^3 + 3x^2)} $$

$$4x^3 - 4x^2 + 5x$$

Divide $4x^3$ by $2x^2$,

$$2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \quad 3x^2 + 2x$$

$$\underline{-(6x^4 - 3x^3 + 3x^2)} $$

$$4x^3 - 4x^2 + 5x$$

Multiply $2x$ by the divisor, $2x^2 - x + 1$,

$$\begin{array}{r}
 3x^2 + 2x \\
 2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \\
 \underline{-(6x^4 - 3x^3 + 3x^2)} \\
 4x^3 - 4x^2 + 5x \\
 \underline{-(4x^3 - 2x^2 + 2x)} \\
 \hline
 \end{array}$$

Subtract,

$$\begin{array}{r}
 3x^2 + 2x \\
 2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \\
 \underline{-(6x^4 - 3x^3 + 3x^2)} \\
 4x^3 - 4x^2 + 5x \\
 \underline{-(4x^3 - 2x^2 + 2x)} \\
 -2x^2 + 3x - 6
 \end{array}$$

Divide $-2x^2$ by $2x^2$,

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \\
 \underline{-(6x^4 - 3x^3 + 3x^2)} \\
 4x^3 - 4x^2 + 5x \\
 \underline{-(4x^3 - 2x^2 + 2x)} \\
 -2x^2 + 3x - 6
 \end{array}$$

Multiply -1 by the divisor, $2x^2 - x + 1$,

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \\
 \underline{-(6x^4 - 3x^3 + 3x^2)} \\
 4x^3 - 4x^2 + 5x \\
 \underline{-(4x^3 - 2x^2 + 2x)} \\
 -2x^2 + 3x - 6 \\
 \underline{-(-2x^2 + x - 1)} \\
 \hline
 \end{array}$$

Subtract,

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 2x^2 - x + 1 \overline{) 6x^4 + x^3 - x^2 + 5x - 6} \\
 \underline{-(6x^4 - 3x^3 + 3x^2)} \\
 4x^3 - 4x^2 + 5x \\
 \underline{-(4x^3 - 2x^2 + 2x)} \\
 -2x^2 + 3x - 6 \\
 \underline{-(-2x^2 + x - 1)} \\
 2x - 5
 \end{array}$$

The highest power of x in the divisor, 2, is now higher than the highest power of x in the dividend, 1. This means we stop dividing. We can now read off our quotient and remainder.

Therefore, the final answer is,

The quotient is $3x^2 + 2x - 1$ and the remainder is $2x - 5$.

3. Solve the inequality $|2x - 1| > 3|x + 2|$. (9709/33/M/J/20 number 1)

$$|2x - 1| > 3|x + 2|$$

Square both sides,

$$(2x - 1)^2 > [3(x + 2)]^2$$

Note: Squaring both sides gets rid of the modulus sign.

Distribute the 3,

$$(2x - 1)^2 > (3x + 6)^2$$

Put both terms on one side,

$$(3x + 6)^2 - (2x - 1)^2 < 0$$

Notice how this is a difference of two squares,

$$a^2 - b^2 = 0$$

Remember that when a difference of two squares is factorised it simplifies to give,

$$(a - b)(a + b) = 0$$

Hence our difference of two squares becomes,

$$(3x + 6)^2 - (2x - 1)^2 < 0$$

$$[(3x + 6) - (2x - 1)][(3x + 6) + (2x - 1)] = 0$$

Expand the brackets inside the square brackets,

$$[3x + 6 - 2x + 1][3x + 6 + 2x - 1] = 0$$

Group like terms and simplify,

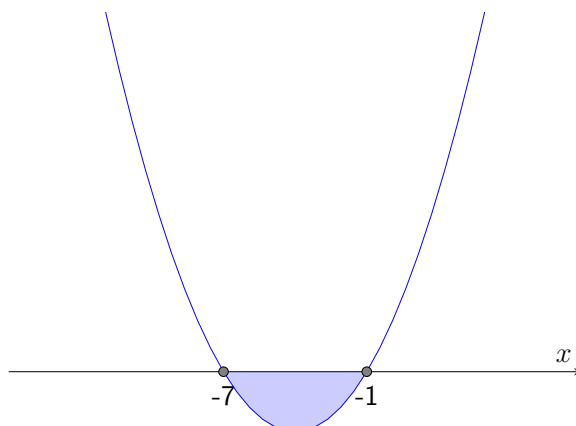
$$(3x - 2x + 6 + 1)(3x + 2x + 6 - 1) = 0$$

$$(x + 7)(5x + 5) = 0$$

That means the roots of our quadratic are,

$$x = -7 \quad x = -1$$

Sketch the graph of the quadratic and determine the required region,



Therefore, the final answer is,

$$-7 < x < -1$$

4. Solve the inequality $2 - 5x > 2|x - 3|$. (9709/31/O/N/20 number 1)

$$2 - 5x > 2|x - 3|$$

Square both sides,

$$(2 - 5x)^2 > [2(x - 3)]^2$$

Note: Squaring both sides gets rid of the modulus sign.

Distribute the 2,

$$(2 - 5x)^2 > (2x - 6)^2$$

Put both terms on one side,

$$(2 - 5x)^2 - (2x - 6)^2 > 0$$

Notice how this is a difference of two squares,

$$a^2 - b^2 = 0$$

Remember that when a difference of two squares is factorised it simplifies to give,

$$(a - b)(a + b) = 0$$

Hence our difference of two squares becomes,

$$(2 - 5x)^2 - (2x - 6)^2 > 0$$

$$[(2 - 5x) - (2x - 6)][(2 - 5x) + (2x - 6)] = 0$$

Expand the brackets inside the square brackets,

$$[2 - 5x - 2x + 6][2 - 5x + 2x - 6] = 0$$

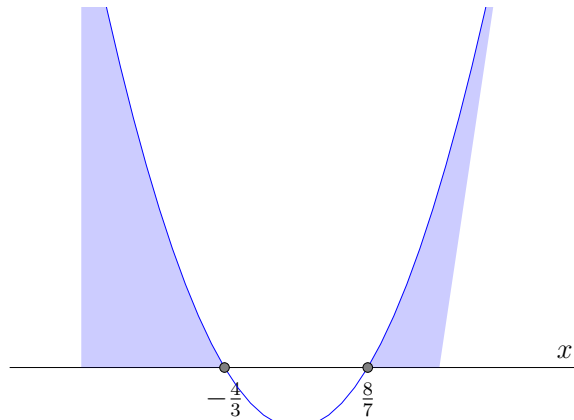
Group like terms and simplify,

$$(8 - 7x)(-4 - 3x) = 0$$

That means the roots of our quadratic are,

$$x = \frac{8}{7} \quad x = -\frac{4}{3}$$

Sketch the graph of the quadratic and determine the required region,



Therefore, the final answer is,

$$x < -\frac{4}{3}$$

Note: If you substitute values greater than $\frac{8}{7}$ into the original inequality, they do not satisfy it. Hence we disregard $x > \frac{8}{7}$.

5. Solve the inequality $2|3x - 1| < |x + 1|$. (9709/31/M/J/21 number 1)

$$2|3x - 1| < |x + 1|$$

Square both sides,

$$[2(3x - 1)]^2 < (x + 1)^2$$

Note: Squaring both sides gets rid of the modulus sign.

Distribute the 2,

$$(6x - 2)^2 < (x + 1)^2$$

Put both terms on one side,

$$(6x - 2)^2 - (x + 1)^2 < 0$$

Notice how this is a difference of two squares,

$$a^2 - b^2 = 0$$

Remember that when a difference of two squares is factorised it simplifies to give,

$$(a - b)(a + b) = 0$$

Hence our difference of two squares becomes,

$$(6x - 2)^2 - (x + 1)^2 < 0$$

$$[(6x - 2) - (x + 1)][(6x - 2) + (x + 1)] = 0$$

Expand the brackets inside the square brackets,

$$[6x - 2 - x - 1][6x - 2 + x + 1] = 0$$

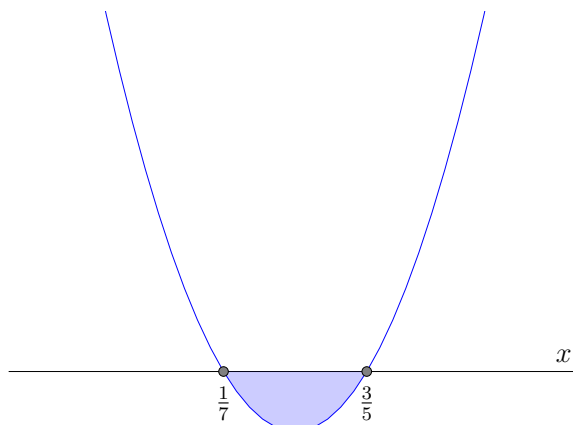
Group like terms and simplify,

$$(5x - 3)(7x - 1) = 0$$

That means the roots of our quadratic are,

$$x = \frac{3}{5} \quad x = \frac{1}{7}$$

Sketch the graph of the quadratic and determine the required region,



Therefore, the final answer is,

$$\frac{1}{7} < x < \frac{3}{5}$$

6. Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. (9709/32/O/N/21 number 2)

$$|3x - a| > 2|x + 2a|$$

Square both sides,

$$(3x - a)^2 > [2(x + 2a)]^2$$

Note: Squaring both sides gets rid of the modulus sign.

Distribute the 2,

$$(3x - a)^2 > (2x + 4a)^2$$

Put both terms on one side,

$$(3x - a)^2 - (2x + 4a)^2 > 0$$

Notice how this is a difference of two squares,

$$a^2 - b^2 = 0$$

Remember that when a difference of two squares is factorised it simplifies to give,

$$(a - b)(a + b) = 0$$

Hence our difference of two squares becomes,

$$(3x - a)^2 - (2x + 4a)^2 > 0$$

$$[(3x - a) - (2x + 4a)][(3x - a) + (2x + 4a)] = 0$$

Expand the brackets inside the square brackets,

$$[3x - a - 2x - 4a][3x - a + 2x + 4a] = 0$$

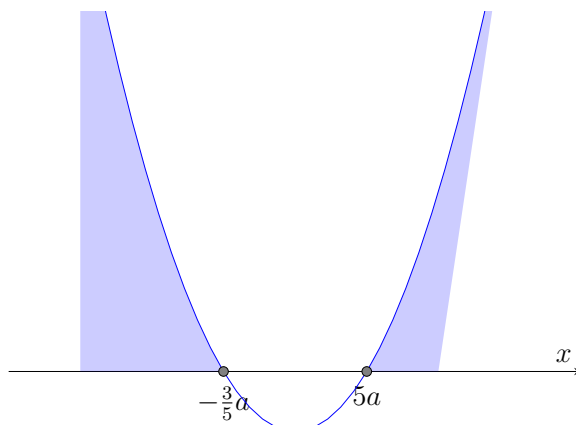
Group like terms and simplify,

$$(x - 5a)(5x + 3a) = 0$$

That means the roots of our quadratic are,

$$x = 5a \quad x = -\frac{3}{5}a$$

Sketch the graph of the quadratic and determine the required region,



Therefore, the final answer is,

$$x < -\frac{3}{5}a \quad x > 5a$$

7. Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. (9709/33/O/N/21 number 1)

$$x^2 - x + 2 \overline{) 2x^4 + 1}$$

Divide $2x^4$ by x^2 ,

$$x^2 - x + 2 \overline{) 2x^4 + 1} \quad \begin{array}{r} 2x^2 \\ \hline \end{array}$$

Multiply $2x^2$ by the divisor, $x^2 - x + 2$,

$$\begin{array}{r} 2x^2 \\ x^2 - x + 2 \overline{) 2x^4 + 1} \\ \underline{-(2x^4 - 2x^3 + 4x^2)} \end{array}$$

Subtract,

$$\begin{array}{r} 2x^2 \\ x^2 - x + 2 \overline{) 2x^4 + 1} \\ \underline{-(2x^4 - 2x^3 + 4x^2)} \\ 2x^3 - 4x^2 + 1 \end{array}$$

Divide $2x^3$ by x^2 ,

$$\begin{array}{r} 2x^2 + 2x \\ x^2 - x + 2 \overline{) 2x^4 + 1} \\ \underline{-(2x^4 - 2x^3 + 4x^2)} \\ 2x^3 - 4x^2 + 1 \end{array}$$

Multiply $2x$ by the divisor, $x^2 - x + 2$,

$$\begin{array}{r}
 \overline{2x^2 + 2x} \\
 x^2 - x + 2 \overline{) 2x^4 + 1} \\
 \underline{-(2x^4 - 2x^3 + 4x^2)} \\
 2x^3 - 4x^2 + 1 \\
 \underline{-(2x^3 - 2x^2 + 4x)} \\
 \hline
 \end{array}$$

Subtract,

$$\begin{array}{r}
 \overline{2x^2 + 2x} \\
 x^2 - x + 2 \overline{) 2x^4 + 1} \\
 \underline{-(2x^4 - 2x^3 + 4x^2)} \\
 2x^3 - 4x^2 + 1 \\
 \underline{-(2x^3 - 2x^2 + 4x)} \\
 -2x^2 - 4x + 1
 \end{array}$$

Divide $-2x^2$ by x^2 ,

$$\begin{array}{r}
 \overline{2x^2 + 2x - 2} \\
 x^2 - x + 2 \overline{) 2x^4 + 1} \\
 \underline{-(2x^4 - 2x^3 + 4x^2)} \\
 2x^3 - 4x^2 + 1 \\
 \underline{-(2x^3 - 2x^2 + 4x)} \\
 -2x^2 - 4x + 1
 \end{array}$$

Multiply -2 by the divisor, $x^2 - x + 2$,

$$\begin{array}{r}
 \overline{2x^2 + 2x - 2} \\
 x^2 - x + 2 \overline{) 2x^4 + 1} \\
 \underline{-(2x^4 - 2x^3 + 4x^2)} \\
 2x^3 - 4x^2 + 1 \\
 \underline{-(2x^3 - 2x^2 + 4x)} \\
 -2x^2 - 4x + 1 \\
 \underline{-(-2x^2 + 2x - 4)} \\
 \hline
 \end{array}$$

Subtract,

$$\begin{array}{r}
 \overline{2x^2 + 2x - 2} \\
 x^2 - x + 2 \overline{) 2x^4 + 1} \\
 \underline{-(2x^4 - 2x^3 + 4x^2)} \\
 2x^3 - 4x^2 + 1 \\
 \underline{-(2x^3 - 2x^2 + 4x)} \\
 -2x^2 - 4x + 1 \\
 \underline{-(-2x^2 + 2x - 4)} \\
 -6x + 5
 \end{array}$$

The highest power of x in the divisor, 2, is now higher than the highest power of x in the dividend, 1. This means we stop dividing. We can now read off our quotient and remainder.

Therefore, the final answer is,

The quotient is $2x^2 + 2x - 2$ and the remainder is $-6x + 5$.

8. Solve the inequality $|2x + 3| > 3|x + 2|$. (9709/32/F/M/22 number 1)

$$|2x + 3| > 3|x + 2|$$

Square both sides,

$$(2x + 3)^2 > [3(x + 2)]^2$$

Note: Squaring both sides gets rid of the modulus sign.

Distribute the 3,

$$(2x + 3)^2 > (3x + 6)^2$$

Put both terms on one side,

$$(3x + 6)^2 - (2x + 3)^2 < 0$$

Notice how this is a difference of two squares,

$$a^2 - b^2 = 0$$

Remember that when a difference of two squares is factorised it simplifies to give,

$$(a - b)(a + b) = 0$$

Hence our difference of two squares becomes,

$$(3x + 6)^2 - (2x + 3)^2 < 0$$

$$[(3x + 6) - (2x + 3)][(3x + 6) + (2x + 3)] = 0$$

Expand the brackets inside the square brackets,

$$[3x + 6 - 2x - 3][3x + 6 + 2x + 3] = 0$$

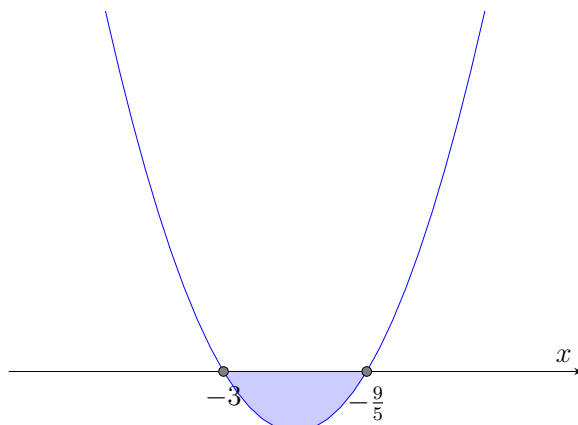
Group like terms and simplify,

$$(x + 3)(5x + 9) = 0$$

That means the roots of our quadratic are,

$$x = -3 \quad x = -\frac{9}{5}$$

Sketch the graph of the quadratic and determine the required region,



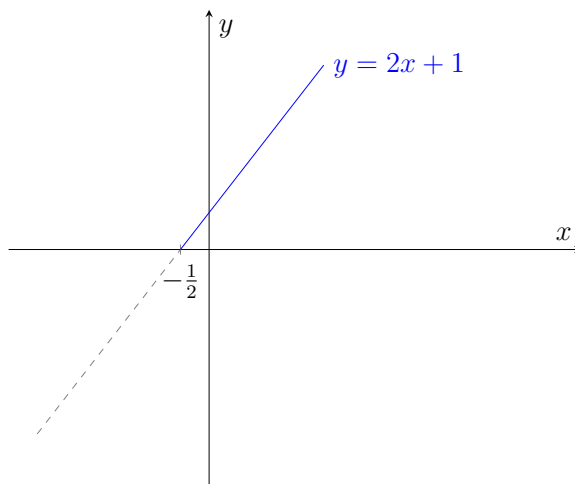
Therefore, the final answer is,

$$-3 < x < -\frac{9}{5}$$

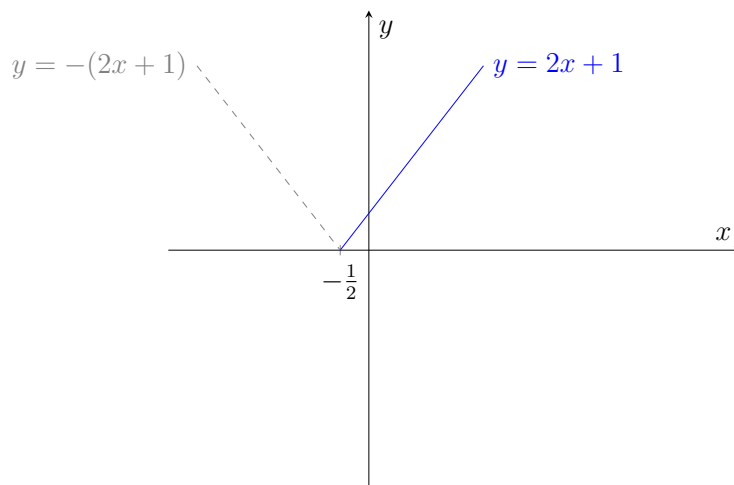
9. (a) Sketch the graph of $y = |2x + 1|$. (9709/31/O/N/22 number 1)

$$y = |2x + 1|$$

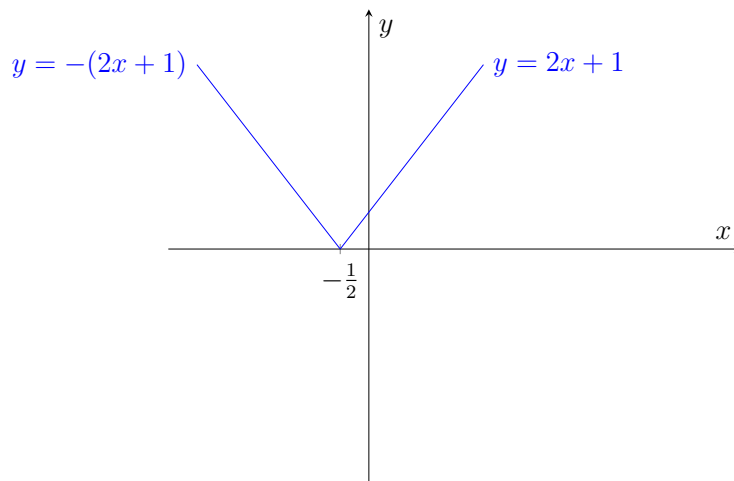
Start by sketching the graph of $y = 2x + 1$,



Reflect the part of the graph that's below the x -axis,



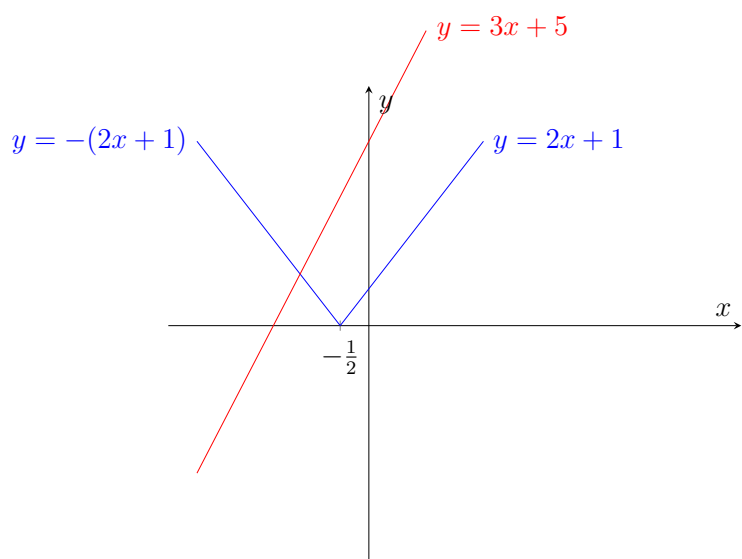
Therefore, the final answer is,



(b) Solve the inequality $3x + 5 < |2x + 1|$.

$$3x + 5 < |2x + 1|$$

Let's start by sketching the graph of $y = 3x + 5$ on the same plane as the graph of $y = |2x + 1|$,



Identify the lines that are intersecting and solve their equations simultaneously,

$$y = -(2x + 1) \quad y = 3x + 5$$

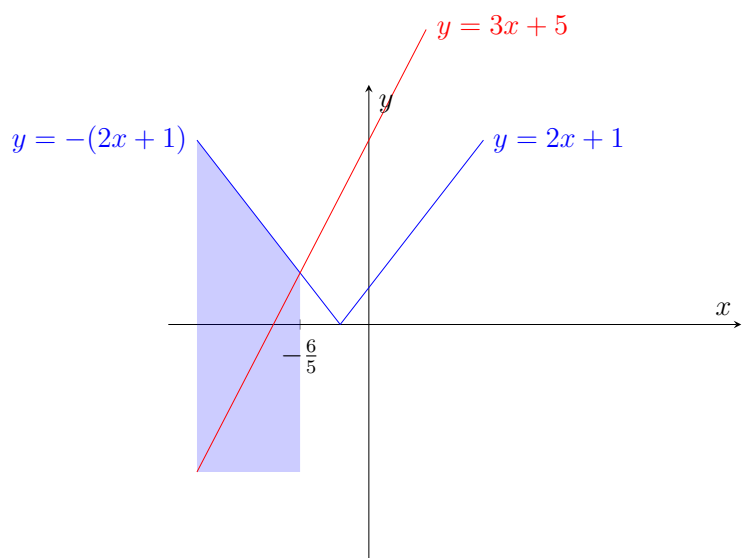
$$-(2x + 1) = 3x + 5$$

$$-2x - 1 = 3x + 5$$

$$5x = -6$$

$$x = -\frac{6}{5}$$

Now we know that $y = 3x + 5$ and $y = |2x + 1|$ intersect when $x = -\frac{6}{5}$. Let's identify the region that satisfies our inequality,



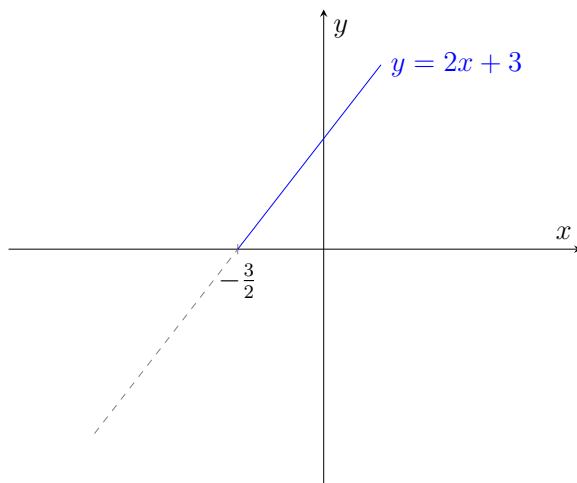
Therefore, the final answer is,

$$x < -\frac{6}{5}$$

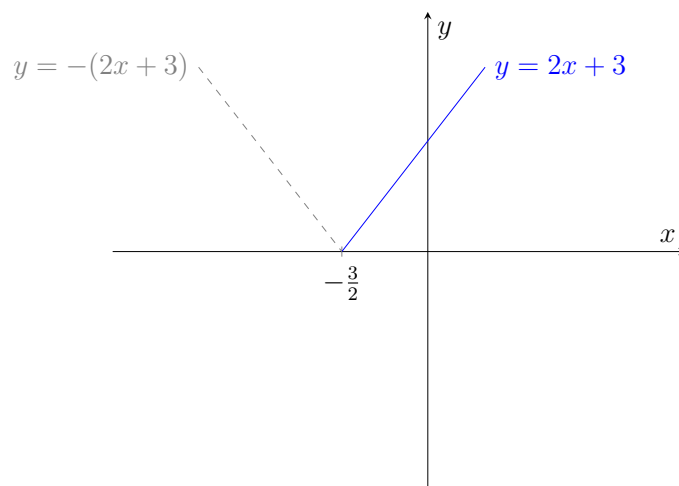
10. (a) Sketch the graph of $y = |2x + 3|$. (9709/31/M/J/23 number 2)

$$y = |2x + 3|$$

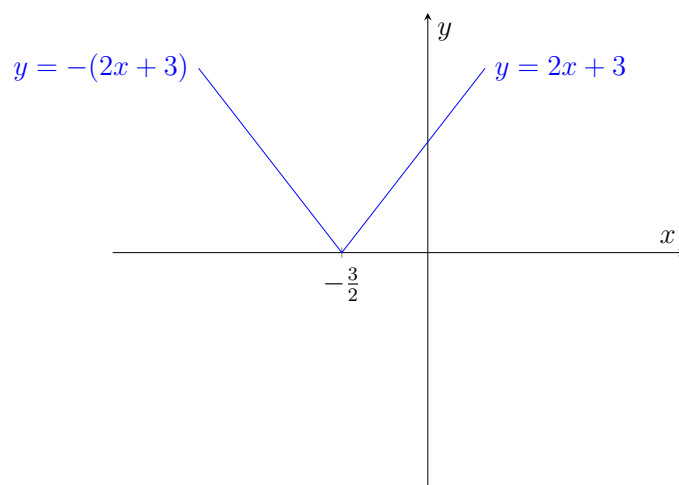
Start by sketching the graph of $y = 2x + 3$,



Reflect the part of the graph that's below the x -axis,



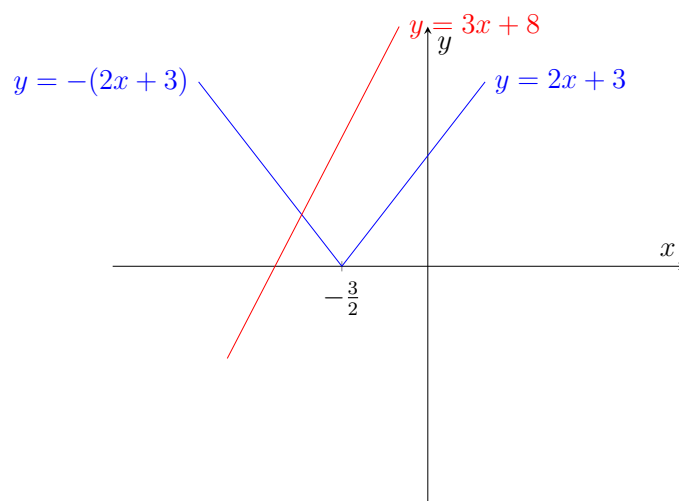
Therefore, the final answer is,



(b) Solve the inequality $3x + 8 > |2x + 3|$.

$$3x + 8 > |2x + 3|$$

Let's start by sketching the graph of $y = 3x + 8$ on the same plane as the graph of $y = |2x + 3|$,



Identify the which lines are intersecting and solve their equations simultaneously,

$$y = -(2x + 3) \quad y = 3x + 8$$

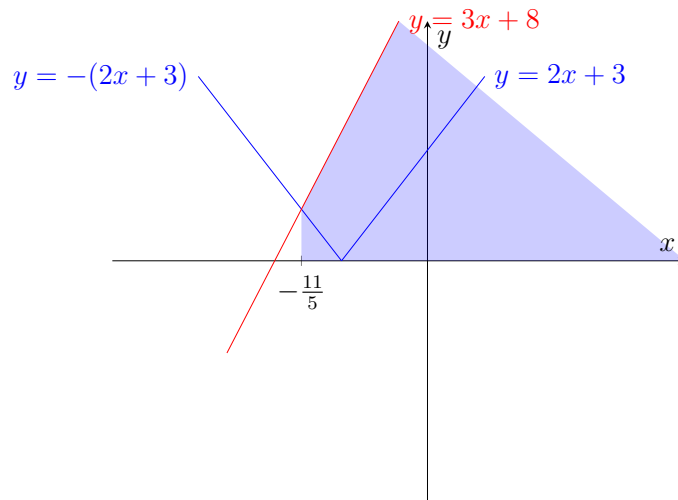
$$-(2x + 3) = 3x + 8$$

$$-2x - 3 = 3x + 8$$

$$5x = -11$$

$$x = -\frac{11}{5}$$

Now we know that $y = 3x + 8$ and $y = |2x + 3|$ intersect when $x = -\frac{11}{5}$. Let's identify the region that satisfies our inequality,



Therefore, the final answer is,

$$x > -\frac{11}{5}$$

11. The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$. Show that $(x + 3)$ is a factor of $p(x)$. (9709/31/M/J/23 number 10a)

$$x^3 + 5x^2 + 31x + 75$$

Using the factor theorem, if $x + 3$ is a factor of $p(x)$ then,

$$p(-3) = 0$$

Let's evaluate $p(-3)$,

$$p(-3) = (-3)^3 + 5(-3)^2 + 31(-3) + 75$$

$$p(-3) = -27 + 45 - 93 + 75$$

$$p(-3) = 120 - 120$$

$$p(-3) = 0$$

Therefore, the final answer is,

$$p(-3) = 0 \text{ therefore, } (x + 3) \text{ is a factor of } p(x).$$

12. Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. (9709/33/M/J/23 number 2)

$$x^2 + x + 3 \overline{) 2x^4 - 27}$$

Divide $2x^4$ by x^2 ,

$$x^2 + x + 3 \overline{) 2x^4 - 27} \quad \begin{array}{r} 2x^2 \\ \hline \end{array}$$

Multiply $2x^2$ by the divisor, $x^2 + x + 3$,

$$\begin{array}{r} 2x^2 \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \end{array}$$

Subtract,

$$\begin{array}{r} 2x^2 \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \\ -2x^3 - 6x^2 - 27 \end{array}$$

Divide $-2x^3$ by x^2 ,

$$\begin{array}{r} 2x^2 - 2x \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \\ -2x^3 - 6x^2 - 27 \end{array}$$

Multiply $-2x$ by the divisor, $x^2 + x + 3$,

$$\begin{array}{r} 2x^2 - 2x \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \\ -2x^3 - 6x^2 - 27 \\ \underline{-(-2x^3 - 2x^2 - 6x)} \end{array}$$

Subtract,

$$\begin{array}{r} 2x^2 - 2x \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \\ -2x^3 - 6x^2 - 27 \\ \underline{-(-2x^3 - 2x^2 - 6x)} \\ -4x^2 + 6x - 27 \end{array}$$

Divide $-4x^2$ by x^2 ,

$$\begin{array}{r} 2x^2 - 2x - 4 \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ \underline{-(2x^4 + 2x^3 + 6x^2)} \\ -2x^3 - 6x^2 - 27 \\ \underline{-(-2x^3 - 2x^2 - 6x)} \\ -4x^2 + 6x - 27 \end{array}$$

Multiply -4 by the divisor, $x^2 + x + 3$,

$$\begin{array}{r} 2x^2 - 2x - 4 \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ - (2x^4 + 2x^3 + 6x^2) \\ \hline -2x^3 - 6x^2 - 27 \\ - (-2x^3 - 2x^2 - 6x) \\ \hline -4x^2 + 6x - 27 \\ - (-4x^2 - 4x - 12) \\ \hline \hline \end{array}$$

Subtract,

$$\begin{array}{r} 2x^2 - 2x - 4 \\ x^2 + x + 3 \overline{) 2x^4 - 27} \\ - (2x^4 + 2x^3 + 6x^2) \\ \hline -2x^3 - 6x^2 - 27 \\ - (-2x^3 - 2x^2 - 6x) \\ \hline -4x^2 + 6x - 27 \\ - (-4x^2 - 4x - 12) \\ \hline 10x - 15 \end{array}$$

The highest power of x in the divisor, 2, is now higher than the highest power of x in the dividend, 1. This means we stop dividing. We can now read off our quotient and remainder.

Therefore, the final answer is,

The quotient is $2x^2 - 2x - 4$ and the remainder is $10x - 15$.