Pure Maths 3

3.3 Trigonometry - Easy

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**A2 Level**
Component:	**Pure Mathematics 3**
Topic:	**3.3 Trigonometry**
Difficulty:	**Easy**

Questions

1. (a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. (9709/31/O/N/20 number 6)

   (b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$.

2. (a) Show that the equation $\tan(\theta + 60°) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$$

   (9709/32/O/N/20 number 4)

   (b) Hence solve the equation $\tan(\theta + 60°) = 2\cot\theta$, for $0° < \theta < 180°$.

3. (a) Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$. (9709/31/M/J/21 number 3)

   (b) Hence solve the equation

$$\cos(x - 30°) = 2\sin(x + 30°)$$

   for $0° < x < 360°$.

4. (a) By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. (9709/33/M/J/21 number 5)

   (b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$,for $0° < \theta < 180°$

5. (a) Express $5\sin x - 3\cos x$ in the form $R\sin x - \alpha$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. (9709/31/O/N/21 number 2)

   (b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$.

6. (a) Show that the equation

$$\cot 2\theta + \cot\theta = 2$$

   can be expressed as a quadratic in $\tan\theta$. (9709/31/O/N/21 number 5)

   (b) Hence solve the equation $\cot 2\theta + \cot\theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places.

7. (a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$. (9709/31/O/N/22 number 6)

   (b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$, for $0° < \theta < 180°$.

8. (a) Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (9709/32/F/M/23 number 6)

   (b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$, for $0 \le x \le \pi$.

9. (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$$

   (9709/31/M/J/23 number 4)

   (b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$, for $0° < \theta < 180°$

10. (a) Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. (9709/33/M/J/23 number 6)

   (b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

   for $0° < \theta < 180°$.

Answers

1. (a) Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. (9709/31/O/N/20 number 6)

$$\sqrt{6}\cos\theta + 3\sin\theta$$

Let's start by finding R,

$$R = \sqrt{\sqrt{6}^2 + 3^2}$$
$$R = \sqrt{15}$$

Now let's find α,

$$\alpha = \arctan\left(\frac{3}{\sqrt{6}}\right)$$
$$\alpha = 50.77°$$

Therefore, the final answer is,

$$\sqrt{6}\cos\theta + 3\sin\theta = \sqrt{15}\cos(\theta - 50.77°)$$

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$.

$$\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$$

Notice how θ has been replaced with $\frac{1}{3}x$,

$$\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x$$

$$\sqrt{6}\cos\theta + 3\sin\theta$$

This means that,

$$\sqrt{15}\cos(\theta - 50.77°)$$
$$\sqrt{15}\cos\left(\frac{1}{3}x - 50.77°\right)$$

Let's solve our equation,

$$\sqrt{15}\cos\left(\frac{1}{3}x - 50.77°\right) = 2.5$$

$$\cos\left(\frac{1}{3}x - 50.77°\right) = \frac{\sqrt{15}}{6}$$

$$\frac{1}{3}x - 50.77° = \arccos\frac{\sqrt{15}}{6}$$
$$P.V = 49.79703411$$
$$\pm P.V + 360n$$

$$\text{At } n = 0$$
$$\pm P.V + 360(0) = (-49.7970, 49.7970)$$
$$\frac{1}{3}x - 50.77 = -49.7970$$
$$x = 2.9^\circ$$
$$\frac{1}{3}x - 50.77 = 49.7970$$
$$x = 301.7^\circ$$

Therefore, the final answer is,

$$x = 2.9^\circ, 301.7^\circ$$

2. (a) Show that the equation $\tan(\theta + 60^\circ) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$$

(9709/32/O/N/20 number 4)

$$\tan(\theta + 60^\circ) = 2\cot\theta$$

Rewrite $\cot\theta$ in terms of $\tan\theta$,

$$\tan(\theta + 60^\circ) = \frac{2}{\tan\theta}$$

Use compound angle formula for $\tan$ to expand,

$$\frac{\tan\theta + \tan 60}{1 - \tan\theta\tan 60} = \frac{2}{\tan\theta}$$

Evaluate $\tan 60$,

$$\frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta} = \frac{2}{\tan\theta}$$

Get rid of the denominators,

$$\tan\theta\left(\tan\theta + \sqrt{3}\right) = 2\left(1 - \sqrt{3}\tan\theta\right)$$

Expand the brackets,

$$\tan^2\theta + \sqrt{3}\tan\theta = 2 - 2\sqrt{3}\tan\theta$$

Put all the terms on one side and simplify,

$$\tan^2\theta + \sqrt{3}\tan\theta + 2\sqrt{3}\tan\theta - 2 = 0$$
$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$$

Therefore, the final answer is,

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$$

(b) Hence solve the equation $\tan(\theta + 60^\circ) = 2\cot\theta$, for $0^\circ < \theta < 180^\circ$.

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$$

Let's solve the quadratic using the quadratic formula,

$$\tan\theta = \frac{-3\sqrt{3} \pm \sqrt{\left(3\sqrt{3}\right)^2 - 4 \times 1 \times -2}}{2(1)}$$

$$\tan\theta = \frac{-3\sqrt{3} - \sqrt{35}}{2} \qquad \tan\theta = \frac{-3\sqrt{3} + \sqrt{35}}{2}$$

Solve the two trig equations,

$$\theta = \arctan\left(\frac{-3\sqrt{3} - \sqrt{35}}{2}\right) \qquad \theta = \arctan\left(\frac{-3\sqrt{3} + \sqrt{35}}{2}\right)$$

$$P.V = -79.7970 \qquad P.V = 19.7970$$

$$-79.7970 + 180 = 100.2030$$

Identify the solutions that are within the given interval,

$$\theta = 19.8°, 100.2°$$

Therefore, the final answer is,

$$\theta = 19.8°, 100.2°$$

3. (a) Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$. (9709/31/M/J/21 number 3)

$$\cos(x - 30°) = 2\sin(x + 30°)$$

Use compound angle formulae to expand,

$$\cos x\cos 30 + \sin x\sin 30 = 2\left(\sin x\cos 30 + \cos x\sin 30\right)$$

Evaluate $\cos 30$ **and** $\sin 30$**,**

$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)$$

Expand the bracket,

$$\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = \sqrt{3}\sin x + \cos x$$

Group like terms and simplify,

$$\sqrt{3}\sin x - \frac{1}{2}\sin x + \cos x - \frac{\sqrt{3}}{2}\cos x = 0$$

$$\frac{-1 + 2\sqrt{3}}{2}\sin x + \frac{2 - \sqrt{3}}{2}\cos x = 0$$

Divide through by $\cos x$,

$$\frac{-1+2\sqrt{3}}{2}\tan x + \frac{2-\sqrt{3}}{2} = 0$$

Make $\tan x$ the subject of the formula,

$$\frac{-1+2\sqrt{3}}{2}\tan x = -\frac{2-\sqrt{3}}{2}$$

$$\tan x = -\frac{2-\sqrt{3}}{2} \times \frac{2}{-1+2\sqrt{3}}$$

$$\tan x = \frac{-(2-\sqrt{3})}{-1+2\sqrt{3}}$$

Factor out a negative sign in the denominator and simplify,

$$\tan x = \frac{-(2-\sqrt{3})}{-(1-2\sqrt{3})}$$

$$\tan x = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$$

Therefore, the final answer is,

$$\tan x = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$$

(b) Hence solve the equation

$$\cos(x-30^\circ) = 2\sin(x+30^\circ)$$

for $0^\circ < x < 360^\circ$.

$$\tan x = \frac{2-\sqrt{3}}{1-2\sqrt{3}}$$

Our given equation can be written in terms of $\tan x$. Let's solve the $\tan x$ equation,

$$x = \arctan\frac{2-\sqrt{3}}{1-2\sqrt{3}}$$

$$P.V = -6.2060$$

$$P.V + 180(1) = 173.7940$$

$$P.V + 180(2) = 353.7940$$

Identify only the solutions that are within the given interval,

$$x = 173.8^\circ, 353.8^\circ$$

Therefore, the final answer is,

$$x = 173.8°, 353.8°$$

4. (a) By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. (9709/33/M/J/21 number 5)

$$\tan 4\theta = \frac{1}{2}\tan\theta$$

Let's start by expanding $\tan(2\theta + 2\theta)$ using the compound angle formula,

$$\frac{\tan 2\theta + \tan 2\theta}{1 - \tan 2\theta \tan 2\theta}$$

$$\frac{2\tan 2\theta}{1 - \tan^2 2\theta}$$

Use the double angle formula to expand $\tan 2\theta$,

$$2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) \div \left[1 - \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2\right]$$

$$\frac{4\tan\theta}{1-\tan^2\theta} \div \left[1 - \frac{4\tan^2\theta}{(1-\tan^2\theta)^2}\right]$$

$$\frac{4\tan\theta}{1-\tan^2\theta} \div \left[\frac{(1-\tan^2\theta)^2 - 4\tan^2\theta}{(1-\tan^2\theta)^2}\right]$$

$$\frac{4\tan\theta}{1-\tan^2\theta} \div \frac{1 - 2\tan^2\theta + \tan^4\theta - 4\tan^2\theta}{(1-\tan^2\theta)^2}$$

$$\frac{4\tan\theta}{1-\tan^2\theta} \div \frac{1 - 6\tan^2\theta + \tan^4\theta}{(1-\tan^2\theta)^2}$$

$$\frac{4\tan\theta}{1-\tan^2\theta} \times \frac{(1-\tan^2\theta)^2}{1 - 6\tan^2\theta + \tan^4\theta}$$

$$\frac{4\tan\theta\,(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$$

This is the complete expansion of $\tan(2\theta + 2\theta)$,

$$\tan(2\theta + 2\theta) = \frac{4\tan\theta\,(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$$

$$\tan 4\theta = \frac{4\tan\theta\,(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$$

Let's equate this to $\frac{1}{2}\tan\theta$ and express it in the required form,

$$\frac{4\tan\theta\,(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta} = \frac{1}{2}\tan\theta$$

Cross multiply,

$$8\tan\theta\left(1-\tan^2\theta\right)=\tan\theta\left(1-6\tan^2\theta+\tan^4\theta\right)$$

Put all the terms on one side,

$$\tan\theta\left(1-6\tan^2\theta+\tan^4\theta\right)-8\tan\theta\left(1-\tan^2\theta\right)=0$$

Factor out $\tan\theta$,

$$\tan\theta\left[\left(1-6\tan^2\theta+\tan^4\theta\right)-8\left(1-\tan^2\theta\right)\right]=0$$
$$\tan\theta\left[1-6\tan^2\theta+\tan^4\theta-8+8\tan^2\theta\right]=0$$
$$\tan\theta\left[\tan^4\theta+2\tan^2\theta-7\right]=0$$
$$\tan\theta=0 \qquad \tan^4\theta+2\tan^2\theta-7=0$$

Therefore, the final answer is,

$$\tan^4\theta+2\tan^2\theta-7=0$$

(b) Hence solve the equation $\tan 4\theta=\frac{1}{2}\tan\theta$,for $0°<\theta<180°$

$$\tan^4\theta+2\tan^2\theta-7=0$$

Let's solve the hidden quadratic using the quadratic formula,

$$\tan^2\theta=\frac{-2\pm\sqrt{2^2-4\times1\times-7}}{2(1)}$$

$$\tan^2\theta=-1\pm2\sqrt{2}$$
$$\tan^2\theta=-1-2\sqrt{2} \qquad \tan^2\theta=-1+2\sqrt{2}$$

Solve the trig equations,

$$\tan\theta=\pm\sqrt{-1-2\sqrt{2}} \qquad \tan\theta=\pm\sqrt{-1+2\sqrt{2}}$$

$$\tan\theta=\text{no solutions} \qquad \tan\theta=\pm\sqrt{-1+2\sqrt{2}}$$

$$\tan\theta=-\sqrt{-1+2\sqrt{2}} \qquad \tan\theta=\sqrt{-1+2\sqrt{2}}$$
$$\theta=\arctan-\sqrt{-1+2\sqrt{2}} \qquad \theta=\arctan\sqrt{-1+2\sqrt{2}}$$
$$P.V=-53.5156 \qquad P.V=53.5156$$
$$-53.5156+180=126.4844$$

Identify the solutions that are within the given interval,

$$\theta=53.5°,126.5°$$

Therefore, the final answer is,

$$\theta = 53.5°, 126.5°$$

5. (a) Express $5\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. (9709/31/O/N/21 number 2)

$$5\sin x - 3\cos x$$

Let's start by finding R,

$$R = \sqrt{5^2 + 3^2}$$
$$R = \sqrt{34}$$

Now let's find α,

$$\alpha = \tan^{-1}\frac{3}{5}$$
$$\alpha = 0.54$$

Therefore, the final answer is,

$$\sqrt{34}\sin(x - 0.54)$$

(b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$.

$$5\sin x - 3\cos x = \sqrt{34}\sin(x - 0.54)$$

Notice that have squared the original equation,

$$(5\sin x - 3\cos x)^2 = 34\sin^2(x - 0.54)$$

Since the graph is now squared it means that it doesn't go below the x-axis, so our least value is 0. The greatest value is 34, since our graph starts at 0 and it is stretched in the y-axis by 34 units.

Therefore, the final answer is,

The greatest value is 34 and the least value is 0.

6. (a) Show that the equation

$$\cot 2\theta + \cot\theta = 2$$

can be expressed as a quadratic in $\tan\theta$. (9709/31/O/N/21 number 5)

$$\cot 2\theta + \cot\theta = 2$$

Rewrite $\cot\theta$ in terms of $\tan\theta$,

$$\frac{1}{2\tan\theta} + \frac{1}{\tan\theta} = 2$$

Use double angle formula for $\tan\theta$,

$$\frac{1}{\frac{2\tan\theta}{1-\tan^2\theta}}+\frac{1}{\tan\theta}=2$$

$$\frac{1-\tan^2\theta}{2\tan\theta}+\frac{1}{\tan\theta}=2$$

Get rid of the denominators,

$$1-\tan^2\theta+2=4\tan\theta$$

$$3-\tan^2\theta=4\tan\theta$$

Put all the terms on one side,

$$\tan^2\theta+4\tan\theta-3=0$$

Therefore, the final answer is,

$$\tan^2\theta+4\tan\theta-3=0$$

(b) Hence solve the equation $\cot 2\theta+\cot\theta=2$, for $0<\theta<\pi$, giving your answers correct to 3 decimal places.

$$\tan^2\theta+4\tan\theta-3=0$$

Let's solve our hidden quadratic using the quadratic formula,

$$\tan\theta=\frac{-4\pm\sqrt{4^2-4\times 1\times -3}}{2(1)}$$

$$\tan\theta=-2\pm\sqrt{7}$$

$$\tan\theta=-2-\sqrt{7}\qquad \tan\theta=-2+\sqrt{7}$$

Solve the trig equations,

$$\theta=\tan^{-1}\left(-2-\sqrt{7}\right)\qquad \theta=\tan^{-1}\left(-2+\sqrt{7}\right)$$

$$P.V=-1.358781\qquad P.V=0.573383$$

$$-1.358781+\pi=1.782812$$

Identify the solutions that are within the given interval,

$$\theta=0.573, 1.783$$

Therefore, the final answer is,

$$\theta=0.573, 1.783$$

7. (a) Prove the identity $\cos 4\theta+4\cos 2\theta+3\equiv 8\cos^4\theta$. (9709/31/O/N/22 number 6)

$$\cos 4\theta+4\cos 2\theta+3\equiv 8\cos^4\theta$$

Let's work from the left-hand side to the right-hand side,

$$\cos 4\theta + 4\cos 2\theta + 3$$

Use the double angle formula for $\cos\theta$ for both $\cos 4\theta$ and $\cos 2\theta$,

$$2\cos^2 2\theta - 1 + 4\left(2\cos^2\theta - 1\right) + 3$$

Use the double angle formula again,

$$2\left(2\cos^2\theta - 1\right)^2 - 1 + 8\cos^2\theta - 4 + 3$$

Expand the quadratic,

$$2\left(4\cos^4\theta - 4\cos^2\theta + 1\right) + 8\cos^2\theta - 2$$

$$8\cos^4\theta - 8\cos^2\theta + 2 + 8\cos^2\theta - 2$$

Simplify like terms,

$$8\cos^4\theta$$

Therefore, the final answer is,

$$\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$$

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$, for $0° < \theta < 180°$.

$$\cos 4\theta + 4\cos 2\theta = 4$$

Using our results from part (a), we can rewrite this as,

$$8\cos^4\theta - 3 = 4$$

Solve the trig equation,

$$\cos^4\theta = \frac{7}{8}$$

$$\cos\theta = \pm\sqrt[4]{\frac{7}{8}}$$

$$\cos\theta = -\sqrt[4]{\frac{7}{8}} \qquad \cos\theta = \sqrt[4]{\frac{7}{8}}$$

$$\theta = \cos^{-1}\left(-\sqrt[4]{\frac{7}{8}}\right) \qquad \theta = \cos^{-1}\left(\sqrt[4]{\frac{7}{8}}\right)$$

$$\theta = 165.3 \qquad \theta = 14.7$$

Therefore, the final answer is,

$$\theta = 14.7°, 165.3°$$

8. (a) Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (9709/32/F/M/23 number 6)

$$5\sin\theta + 12\cos\theta$$

Let's start by finding R,

$$R = \sqrt{5^2 + 12^2}$$
$$R = 13$$

Now let's find α,

$$\alpha = \tan^{-1}\left(\frac{5}{12}\right)$$
$$\alpha = 0.395$$

Therefore, the final answer is,

$$13\cos(\theta - 0.395)$$

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$, for $0 \leq x \leq \pi$.

$$5\sin 2x + 12\cos 2x = 6$$

Notice how the θ has been replaced by $2x$,

$$5\sin \textcolor{blue}{2x} + 12\cos \textcolor{blue}{2x}$$

$$5\sin \textcolor{blue}{\theta} + 12\cos \textcolor{blue}{\theta}$$

This means we can rewrite our problem as,

$$13\cos(2x - 0.395) = 6$$

Solve the trig equation,

$$\cos(2x - 0.395) = \frac{6}{13}$$
$$2x - 0.395 = \cos^{-1}\left(\frac{6}{13}\right)$$
$$P.V = 1.091068$$
$$-1.091068 + 2\pi = 5.1921$$
$$2x - 0.395 = 1.091068$$
$$x = 0.743$$
$$2x - 0.395 = 5.1921$$
$$x = 2.79$$

Therefore, the final answer is,

$$x = 0.743, 2.79$$

9. (a) Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$$

(9709/31/M/J/23 number 4)

$$\sin 2\theta + \cos 2\theta = 2\sin^2\theta$$

Use double angle formulae to simplify,

$$2\sin\theta\cos\theta + \cos^2\theta - \sin^2\theta = 2\sin^2\theta$$

Put all the terms on one side and simplify,

$$\cos^2\theta + 2\sin\theta\cos\theta - \sin^2\theta - 2\sin^2\theta = 0$$
$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$$

Therefore, the final answer is,

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$$

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$, for $0° < \theta < 180°$

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0$$

We can factorise this equation,

$$(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$$

Equate each bracket to 0,

$$\cos\theta - \sin\theta = 0 \qquad \cos\theta + 3\sin\theta = 0$$

Divide through by $\cos\theta$,

$$1 - \tan\theta = 0 \qquad 1 + 3\tan\theta = 0$$

Solve the trig equations,

$$\tan\theta = 1 \qquad \tan\theta = -\frac{1}{3}$$
$$\theta = \tan^{-1} 1 \qquad \theta = \tan^{-1} -\frac{1}{3}$$
$$\theta = 45 \qquad P.V = -18.4349$$
$$-18.4349 + 180 = 161.5651$$

Identify the solutions that are within the given interval,

$$\theta = 45°, 161.6°$$

Therefore, the final answer is,

$$\theta = 45°, 161.6°$$

10. (a) Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. (9709/33/M/J/23 number 6)

$$3\cos x + 2\cos(x - 60°)$$

Use the compound angle formula to expand,

$$3\cos x + 2\left(\cos x \cos 60 + \sin x \sin 60\right)$$

Evaluate $\cos 60$ **and** $\sin 60$**,**

$$3\cos x + 2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$
$$3\cos x + \cos x + \sqrt{3}\sin x$$
$$4\cos x + \sqrt{3}\sin x$$

Now let's find R**,**

$$R = \sqrt{4^2 + \left(\sqrt{3}\right)^2}$$
$$R = \sqrt{19}$$

Now let's find α**,**

$$\alpha = \tan^{-1}\frac{\sqrt{3}}{4}$$
$$\alpha = 23.41$$

Therefore, the final answer is,

$$\sqrt{19}\cos(x - 23.41)$$

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos\left(2\theta - 60^\circ\right) = 2.5$$

for $0^\circ < \theta < 180^\circ$.

$$3\cos 2\theta + 2\cos\left(2\theta - 60^\circ\right) = 2.5$$

Notice how the x **has been replaced with** 2θ**,**

$$3\cos 2\theta + 2\cos\left(2\theta - 60^\circ\right)$$
$$3\cos x + 2\cos\left(x - 60^\circ\right)$$

This means we can rewrite our equation as,

$$\sqrt{19}\cos(2\theta - 23.41) = 2.5$$

Solve the trig equation,

$$\cos(2\theta - 23.41) = \frac{2.5}{\sqrt{19}}$$
$$2\theta - 23.41 = \cos^{-1}\frac{2.5}{\sqrt{19}}$$
$$P.V = 55.0026$$
$$-55.0026 + 360 = 304.9974$$
$$2\theta - 23.41 = 55.0026$$
$$\theta = 39.2^\circ$$
$$2\theta - 23.41 = 304.9974$$
$$\theta = 164.2^\circ$$

Therefore, the final answer is,

$$\theta = 39.2^\circ, 164.2^\circ$$