Pure Maths 3

3.9 Complex Numbers - Easy

GCEALevelMaths9709

Subject:	**Mathematics**
Syllabus Code:	**9709**
Level:	**A2 Level**
Component:	**Pure Mathematics 3**
Topic:	**3.9 Complex Numbers**
Difficulty:	**Easy**

Questions

1. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z-1+i| \leq 1$. (9709/31/O/N/20 number 2)

2. (a) Verify that $-1+\sqrt{5}i$ is a root of the equation $2x^3+x^2+6x-18=0$. (9709/31/O/N/20 number 7)

   (b) Find the other roots of this equation.

3. The complex number u is defined by
$$u = \frac{7+i}{1-i}$$
(9709/32/O/N/20 number 6)

   (a) Express u in the form $x+iy$, where x and y are real.

   (b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7+i$ and $1-i$ respectively.

   (c) By considering the arguments of $7+i$ and $1-i$, show that
$$\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{7} + \frac{1}{4}\pi$$

4. (a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. (9709/31/M/J/21 number 5)

   In the Argand diagram with the origin O, the roots of this equation are represented by the distinct points A and B.

   (b) Given that A and B lie on the imaginary axis, find a relation between p and q.

   (c) Given instead that triangle OAB is equilateral, express q in terms of p.

5. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z+1-i| \leq 1$ and $\arg(z-1) \leq \frac{3}{4}\pi$. (9709/32/M/J/21 number 2)

6. (a) Given the complex numbers $u = a+ib$ and $w = c+id$, where a, b, c and d are real, prove that $(u+w)^* = u^* + w^*$. (9709/32/O/N/21 number 3)

   (b) Solve the equation $(z+2+i)^* + (2+i)z = 0$, giving your answer in the form $x+iy$ where x and y are real.

7. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z+2-3i| \leq 2$ and $\arg z \leq \frac{3}{4}\pi$. (9709/32/F/M/22 number 2)

8. The complex number $3-i$ is denoted by u. (9709/33/M/J/22 number 5)

   (a) Show, on an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and u^*-u respectively.

   State the type of quadrilateral formed by the points O, A, B and C.

   (b) Express $\frac{u^*}{u}$ in the form $x+iy$, where x and y are real.

   (c) By considering the argument of $\frac{u^*}{u}$, or otherwise, prove that $\tan^{-1}\left(\frac{3}{4}\right) = 2\tan^{-1}\left(\frac{1}{3}\right)$.

9. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z| \leq 3$, $\Re z \geq -2$ and $\frac{1}{4}\pi \leq \arg z \leq \pi$. (9709/31/O/N/22 number 2)

10. The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$. (9709/31/O/N/22 number 5)

(a) Find $\frac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ. (9709/31/O/N/22 number 5)

(b) State the least positive integer n such that both $\Im w^n = 0$ and $\Re w^n > 0$.

11. (a) On an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \leq \arg(z - 1 - 2i) \leq \frac{1}{3}\pi$ and $\Re z \leq 3$. (9709/32/F/M/23 number 2)

    (b) Calculate the least value of $\arg z$ for points in the region from (a). Give your answer in radians correct to 3 decimal places.

12. (a) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. (9709/32/M/J/23 number 3)

    (b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form.

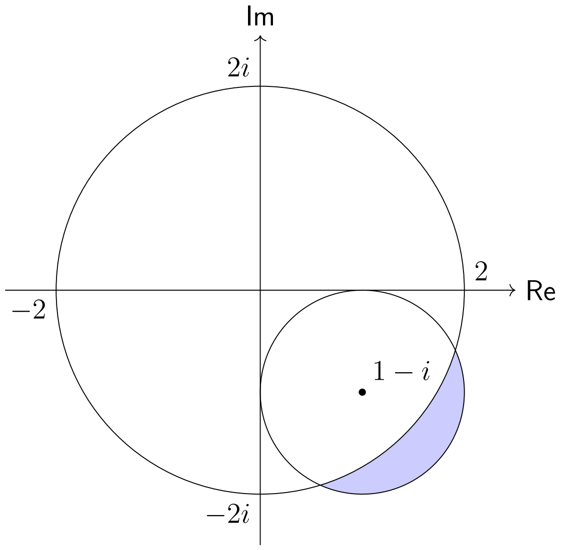
Answers

1. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. (9709/31/O/N/20 number 2)

Let's rewrite the second inequality,

$$|z| \geq 2 \quad |z - (1 - i)| \leq 1$$

Draw the region that satisfies both inequalities,



2. (a) Verify that $-1 + \sqrt{5}i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. (9709/31/O/N/20 number 7)

$$2x^3 + x^2 + 6x - 18 = 0$$

Substitute the root into the equation,

$$2(-1 + \sqrt{5}i)^3 + (-1 + \sqrt{5}i)^2 + 6(-1 + \sqrt{5}i) - 18 = 0$$

Expand,

$$2(-1 + \sqrt{5}i)^3 + 1 - 2\sqrt{5}i + 5i^2 - 6 + 6\sqrt{5}i - 18 = 0$$
$$2(-1 + \sqrt{5}i)^3 + 1 - 2\sqrt{5}i + 5(-1) - 6 + 6\sqrt{5}i - 18 = 0$$
$$2(-1 + \sqrt{5}i)^3 + 1 - 2\sqrt{5}i - 5 - 6 + 6\sqrt{5}i - 18 = 0$$
$$2(-1 + \sqrt{5}i)^3 + 4\sqrt{5}i - 28 = 0$$

Let's use binomial expansion to expand the cubic,

$$(-1 + \sqrt{5}i)^3$$

$$(-1)^3 + \binom{3}{1}(-1)^2(\sqrt{5}i) + \binom{3}{2}(-1)(\sqrt{5}i)^2 + \binom{3}{3}(\sqrt{5}i)^3$$

$$-1+3\sqrt{5}i-3(5i^2)+5\sqrt{5}i^3$$
$$-1+3\sqrt{5}i-3(5(-1))+5\sqrt{5}i^2\times i$$
$$-1+15+3\sqrt{5}i+5\sqrt{5}(-1)i$$
$$14+3\sqrt{5}i-5\sqrt{5}i$$
$$14-2\sqrt{5}i$$

Let's go back to our equation,

$$214-2\sqrt{5}i+4\sqrt{5}i-28=0$$

$$28-4\sqrt{5}i+4\sqrt{5}i-28=0$$

Therefore, the final answer is,

$-1+\sqrt{5}i$ is indeed a root of the equation.

(b) Find the other roots of this equation.

$$2x^3+x^2+6x-18=0$$

The complex conjugate of $-1+\sqrt{5}i$ is also a root of the equation,

$$-1-\sqrt{5}i$$

Now let's use these two roots to find the quadratic factor of our polynomial,

$$x=-1+\sqrt{5}i \quad x=-1-\sqrt{5}i$$

$$(x-(-1+\sqrt{5}i))(x-(-1-\sqrt{5}i))=0$$

Expand the brackets,

$$x^2-(-1-\sqrt{5}i)x-(-1+\sqrt{5}i)x+(-1+\sqrt{5}i)(-1-\sqrt{5}i)=0$$

$$x^2+x+\sqrt{5}ix+x-\sqrt{5}ix+1-5i^2=0$$
$$x^2+2x+1-5(-1)=0$$
$$x^2+2x+1+5=0$$
$$x^2+2x+6=0$$

We have found the quadratic factor of our polynomial. Let's compare coefficients to find the linear factor,

$$2x^3+x^2+6x-18=(x^2+2x+6)(ax+b)$$

$$2x^3=x^2\times ax \qquad -18=6b$$
$$2x^3=ax^3 \qquad b=-3$$
$$a=2$$

This means that our linear factor is,

$$2x - 3$$

The solution we get from the linear factor is,

$$2x - 3 = 0$$

$$x = \frac{3}{2}$$

Therefore, the final answer is,

$$x = -1 - \sqrt{5}i \quad x = \frac{3}{2}$$

3. The complex number u is defined by

$$u = \frac{7+i}{1-i}$$

(9709/32/O/N/20 number 6)

(a) Express u in the form $x + iy$, where x and y are real.

$$u = \frac{7+i}{1-i}$$

Multiply both the numerator and denominator by the complex conjugate of the denominator, $1 + i$**,**

$$\frac{(7+i)(1+i)}{(1-i)(1+i)}$$

$$\frac{7+i^2+8i}{1-i^2}$$

$$\frac{7+(-1)+8i}{1-(-1)}$$

$$\frac{6+8i}{2}$$

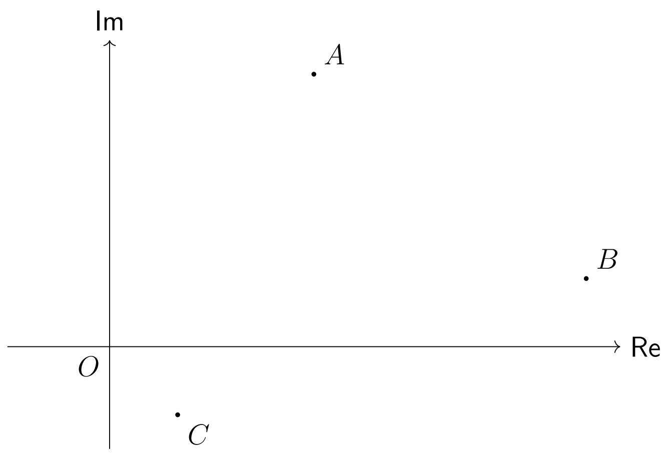
$$3+4i$$

Therefore, the final answer is,

$$u = 3 + 4i$$

(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7+i$ and $1-i$ respectively.

Sketch the argand diagram,



(c) By considering the arguments of $7+i$ and $1-i$, show that

$$\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{7} + \frac{1}{4}\pi$$

The argument of $7+i$ is,

$$\arg(7+i) = \tan^{-}\frac{1}{7}$$

The argument of $1-i$ is,

$$\arg(1-i) = -\tan^{-1} 1$$

$$\arg(1-i) = -\frac{1}{4}\pi$$

The argument of $\frac{7+i}{1-i}$, which we deduced to be $3+4i$, is,

$$\arg\left(\frac{7+i}{1-i}\right) = \tan^{-1}\frac{4}{3}$$

We can say that,

$$\arg\left(\frac{7+i}{1-i}\right) = \arg(7+i) - \arg(1-i)$$

$$\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{7} - \left(-\frac{1}{4}\pi\right)$$

$$\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{7} + \frac{1}{4}\pi$$

Therefore, the final answer is,

$$\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{7} + \frac{1}{4}\pi$$

4. (a) Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. (9709/31/M/J/21 number 5)

$$z^2 - 2piz - q = 0$$

Let's use the quadratic formula to solve our quadratic equation,

$$z = \frac{2pi \pm \sqrt{(2pi)^2 - 4 \times 1 \times -q}}{2(1)}$$

$$z = \frac{2pi \pm \sqrt{4p^2i^2 + 4q}}{2}$$

$$z = \frac{2pi \pm \sqrt{4p^2(-1) + 4q}}{2}$$

$$z = \frac{2pi \pm \sqrt{-4p^2 + 4q}}{2}$$

$$z = \frac{2pi \pm \sqrt{4\left(q - p^2\right)}}{2}$$

$$z = \frac{2pi \pm \sqrt{4}\sqrt{(q - p^2)}}{2}$$

$$z = \frac{2pi \pm 2\sqrt{(q - p^2)}}{2}$$

$$z = pi \pm \sqrt{(q - p^2)}$$

Therefore, the final answer is,

$$z = pi \pm \sqrt{(q - p^2)}$$

In the Argand diagram with the origin O, the roots of this equation are represented by the distinct points A and B.

(b) Given that A and B lie on the imaginary axis, find a relation between p and q.

If the roots of this equation lie on the imaginary axis that means that our quadratic has imaginary roots i.e no real roots,

$$b^2 - 4ac < 0$$

Let's identify a, b and c from our equation,

$$a = 1 \quad b = -2pi \quad c = -q$$

Substitute into the discriminant,

$$(-2pi)^2 - 4(1)(-q) < 0$$

$$4p^2i^2 + 4q < 0$$

$$-4p^2 + 4q < 0$$

$$4q < 4p^2$$

$$q < p^2$$

Therefore, the final answer is,

$$q < p^2$$

(c) Given instead that triangle OAB is equilateral, express q in terms of p.

$$A = pi + \sqrt{(q - p^2)} \quad B = pi - \sqrt{(q - p^2)}$$

If OAB is equilateral then the argument of B is $60°$,

$$\arg B = \tan^{-1}\left(-\frac{p}{\sqrt{(q - p^2)}}\right)$$

$$60 = \tan^{-1}\left(-\frac{p}{\sqrt{(q - p^2)}}\right)$$

$$\tan(60) = -\frac{p}{\sqrt{(q - p^2)}}$$

$$-\frac{p}{\sqrt{(q - p^2)}} = \sqrt{3}$$

Square both sides,

$$\frac{p^2}{q - p^2} = 3$$

Get rid of the denominator,

$$p^2 = 3\left(q - p^2\right)$$

$$p^2 = 3q - 3p^2$$

$$3q = 4p^2$$

$$q = \frac{4}{3}p^2$$

Therefore, the final answer is,

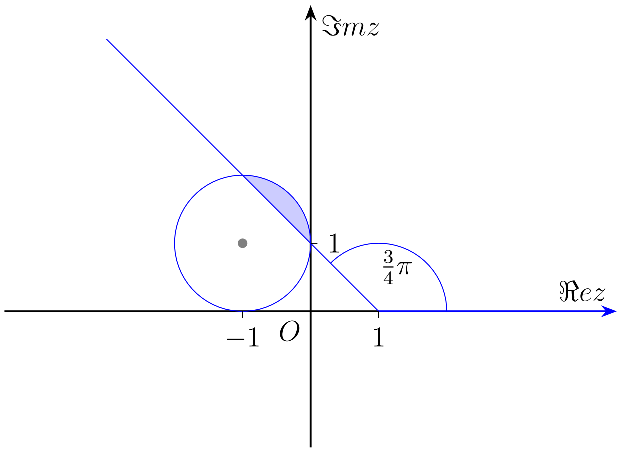
$$q = \frac{4}{3}p^2$$

5. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z + 1 - i| \leq 1$ and $\arg(z - 1) \leq \frac{3}{4}\pi$. (9709/32/M/J/21 number 2)

Factor out a negative sign in the inequalities to see the coordinates,

$$|z - (-1 + i)| \leq 1 \quad \arg(z - (1 + 0i)) \leq \frac{3}{4}\pi$$

Sketch the argand diagram,



6. (a) Given the complex numbers $u = a + ib$ and $w = c + id$, where a, b, c and d are real, prove that $(u + w)^* = u^* + w^*$. (9709/32/O/N/21 number 3)

Let's start by finding u^* and w^*,

$$u^* = a - ib \quad w^* = c - id$$

Let's add them together,

$$u^* + w^* = a - ib + c - id$$

$$u^* + w^* = (a + c) - (b + d)i$$

Now let's find $(u + w)^*$,

$$u + w = a + ib + c + id$$

$$u + w = (a + c) + (b + d)i$$

$$(u + w)^* = (a + c) - (b + d)i$$

Therefore, the final answer is,

$$u + w)^* = u^* + w^*$$

(b) Solve the equation $(z + 2 + i)^* + (2 + i)z = 0$, giving your answer in the form $x + iy$ where x and y are real.

$$(z + 2 + i)^* + (2 + i)z = 0$$

In part (a), we proved that,

$$u + w)^* = u^* + w^*$$

This means that,

$$(z + 2 + i)^* = z^* + (2 + i)^*$$

$$(z + 2 + i)^* = z * +2 - i$$

Substitute into the equation,

$$z * +2 - i + (2+i)z = 0$$

Substitute z with $x+iy$ and expand the brackets,

$$(x+iy) * +2 - i + (2+i)(x+iy) = 0$$

$$x - iy + 2 - i + 2x + 2iy + ix + i^2y = 0$$

Simplify like terms and remember that $i^2 = -1$,

$$3x + iy - y + 2 - i + ix = 0$$

Group the real parts together and the imaginary together,

$$3x + 2 - y + iy + ix - i = 0$$

$$3x + 2 - y + i(y + x - 1) = 0$$

Equate the real part to 0 and equate the imaginary part to 0,

$$3x + 2 - y = 0 \quad y + x - 1 = 0$$

Solve the two equations simultaneously,

$$y = 3x + 2$$

$$y + x - 1 = 0$$
$$3x + 2 + x - 1 = 0$$
$$4x + 1 = 0$$
$$4x = -1$$
$$x = -\frac{1}{4}$$

Evaluate y,

$$y = 3\left(-\frac{1}{4}\right) + 2$$
$$y = \frac{5}{4}$$

Therefore, the final answer is,

$$z = -\frac{1}{4} + \frac{5}{4}i$$

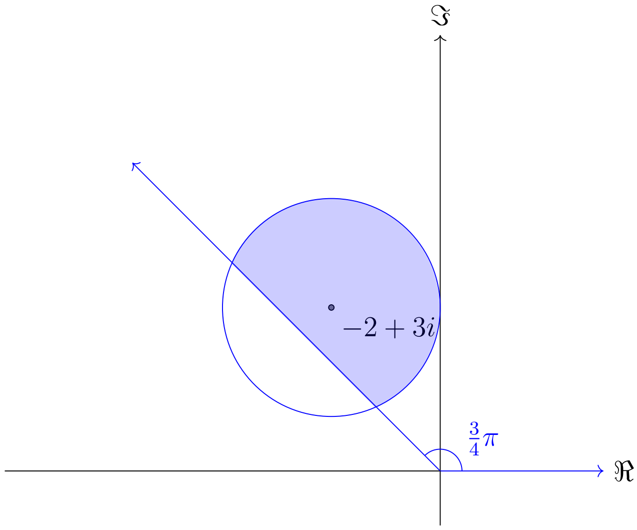
7. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z+2-3i| \leq 2$ and $\arg z \leq \frac{3}{4}\pi$. (9709/32/F/M/22 number 2)

$$|z+2-3i| \leq 2 \quad \arg z \leq \frac{3}{4}\pi$$

Rewrite the first inequality to make the coordinates clear,

$$|z - (-2 + 3i)| \leq 2 \qquad \arg z \leq \frac{3}{4}\pi$$

Sketch the argand diagram,

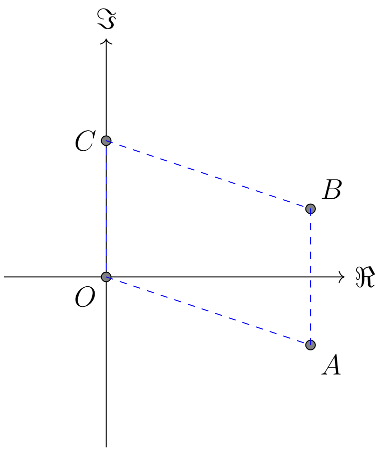


8. The complex number $3 - i$ is denoted by u. (9709/33/M/J/22 number 5)

   (a) Show, on an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively.
   State the type of quadrilateral formed by the points O, A, B and C.

$$u = 3 - i \quad u^* = 3 + i \quad u * -u = 2i$$

Let's sketch the Argand diagram,



If we join the points a parallelogram is formed.

Therefore, the final answer is,

$$OABC \text{ is a parallelogram.}$$

(b) Express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real.

$$\frac{u^*}{u} = \frac{3+i}{3-i}$$

Multiply both the numerator and the denominator by u^*,

$$\frac{(3+i)(3+i)}{(3-i)(3+i)}$$

$$\frac{9+6i+i^2}{9-i^2}$$

$$\frac{9+6i-1}{9-(-1)}$$

$$\frac{8+6i}{10}$$

$$\frac{4}{5}+\frac{3}{5}i$$

Therefore, the final answer is,

$$\frac{u^*}{u} = \frac{4}{5}+\frac{3}{5}i$$

(c) By considering the argument of $\frac{u^*}{u}$, or otherwise, prove that $\tan^{-1}\left(\frac{3}{4}\right) = 2\tan^{-1}\left(\frac{1}{3}\right)$.

$$\frac{u^*}{u} = \frac{4}{5}+\frac{3}{5}i$$

The argument of $\frac{u^*}{u}$ is,

$$\arg\left(\frac{u^*}{u}\right) = \tan^{-1}\left(\frac{3}{5} \div \frac{4}{5}\right)$$

$$\arg\left(\frac{u^*}{u}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

Let's find the argument of u,

$$\arg u = -\tan^{-1}\frac{1}{3}$$

Let's find the argument of u^*,

$$\arg u^* = \tan^{-1}\frac{1}{3}$$

We can write the argument of $\frac{u^*}{u}$ as,

$$\arg\left(\frac{u^*}{u}\right) = \arg u* - \arg u$$

Substitute,

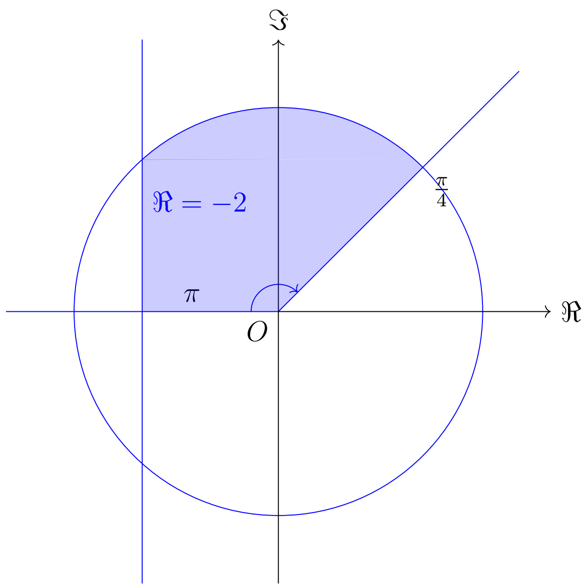
$$\tan^{-1}\frac{3}{4} = \tan^{-1}\frac{1}{3} - \left(-\tan^{-1}\frac{1}{3}\right)$$
$$\tan^{-1}\frac{3}{4} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3}$$
$$\tan^{-1}\frac{3}{4} = 2\tan^{-1}\frac{1}{3}$$

Therefore, the final answer is,

$$\tan^{-1}\frac{3}{4} = 2\tan^{-1}\frac{1}{3}$$

9. On a sketch of an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $|z| \leq 3$, $\Re z \geq -2$ and $\frac{1}{4}\pi \leq \arg z \leq \pi$. (9709/31/O/N/22 number 2)

Sketch the argand diagram,



10. The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$. (9709/31/O/N/22 number 5)

    (a) Find $\frac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ. (9709/31/O/N/22 number 5)

$$u = 2e^{\frac{1}{4}\pi i} \quad w = 3e^{\frac{1}{3}\pi i}$$

Let's start by squaring u,

$$u^2 = \left(2e^{\frac{1}{4}\pi i}\right)$$
$$u^2 = 2^2 e^{2\left(\frac{1}{4}\pi i\right)}$$
$$u^2 = 4e^{\frac{1}{2}\pi i}$$

Now let's find $\frac{u^2}{w}$,

$$\frac{u^2}{w} = \frac{4e^{\frac{1}{2}\pi i}}{3e^{\frac{1}{3}\pi i}}$$

$$\frac{u^2}{w} = \frac{4}{3}e^{\frac{1}{6}\pi i}$$

Therefore, the final answer is,

$$\frac{u^2}{w} = \frac{4}{3}e^{\frac{1}{6}\pi i}$$

(b) State the least positive integer n such that both $\Im w^n = 0$ and $\Re w^n > 0$.

$$w = 3e^{\frac{1}{3}\pi i}$$

Let's start by evaluating w^n,

$$w^n = \left(3e^{\frac{1}{3}\pi i}\right)^n$$

Distribute the power n,

$$w^n = 3^n e^{\frac{1}{3}n\pi i}$$

Now let's move from the euler form to the polar form,

$$re^{i\theta} \equiv r(\cos\theta + i\sin\theta)$$

$$3^n e^{\frac{1}{3}n\pi i} = 3^n\left(\cos\left(\frac{1}{3}n\pi\right) + i\sin\left(\frac{1}{3}n\pi\right)\right)$$

Expand the bracket,

$$3^n\cos\left(\frac{1}{3}n\pi\right) + 3^n\sin\left(\frac{1}{3}n\pi\right)i$$

Now that we have our complex number in the form $x + iy$, identify the real part and the imaginary part. We were told that $\Im w^n = 0$ and $\Re w^n > 0$,

$$3^n\sin\left(\frac{1}{3}n\pi\right) = 0 \qquad 3^n\cos\left(\frac{1}{3}n\pi\right) > 0$$

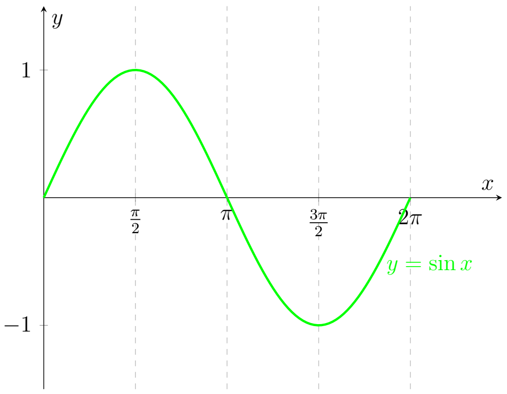
We can write this as,

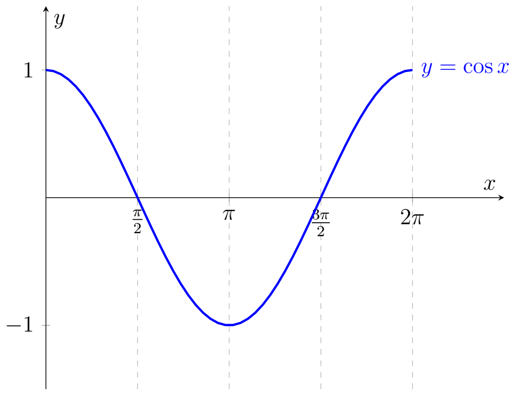
$$3^n = 0 \quad \sin\left(\frac{1}{3}n\pi\right) = 0 \qquad 3^n > 0 \quad \cos\left(\frac{1}{3}n\pi\right) > 0$$

$3^n = 0$ **has no solutions. So this becomes,**

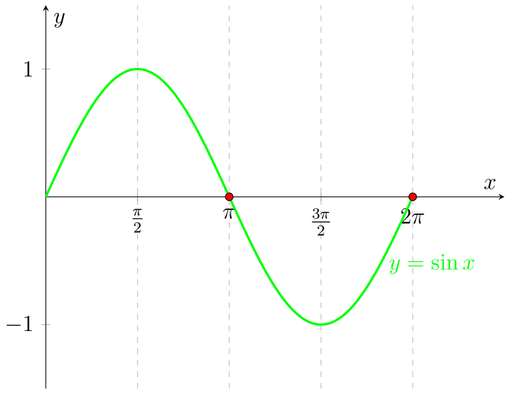
$$\sin\left(\frac{1}{3}n\pi\right) = 0 \qquad \cos\left(\frac{1}{3}n\pi\right) > 0$$

Let's sketch the graphs of $y = \sin x$ and $y = \cos x$,

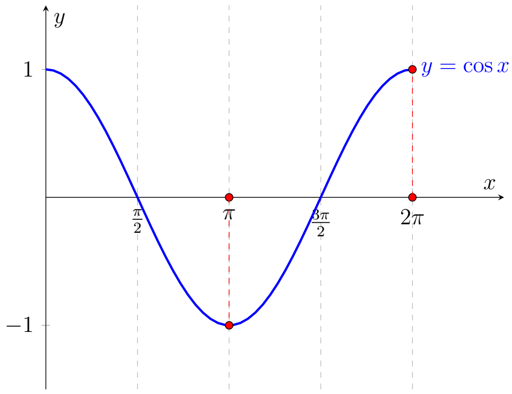




For the graph of $y = \sin x$, identify all the points where $y = 0$ i.e $\sin\left(\frac{1}{3}n\pi\right) = 0$,



These points satisfy the equation $\Im w^n = 0$, since this green graph represents our imaginary part. Now let's check if any of those two points satisfy $\Re w^n > 0$ i.e $\cos\left(\frac{1}{3}n\pi\right) > 0$,



Note: We're checking to see if the graph is below or above the x-axis at these points. And we want the point where the graph is above the x-axis.

The point $x = 2\pi$, satisfies both $\Im w^n = 0$ and $\Re w^n > 0$. Now let's evaluate n when $x = 2\pi$,

$$\frac{1}{3}\pi n = 2\pi$$

$$n = 2 \times 3$$
$$n = 6$$

Note: We were using the functions $y = \sin x$ and $y = \cos x$ for demonstration purposes but remember that for our original functions we were dealing with $\frac{1}{3}\pi n$.

Therefore, the final answer is,

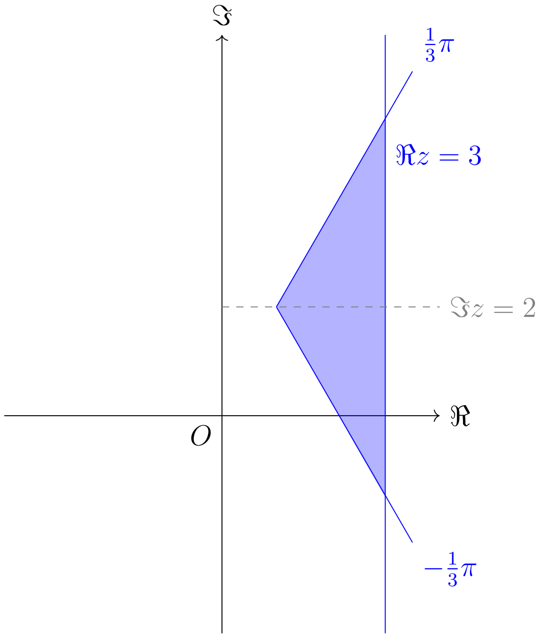
$$n = 6$$

11. (a) On an Argand diagram, shade the region whose points represents the complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \leq \arg(z - 1 - 2i) \leq \frac{1}{3}\pi$ and $\Re z \leq 3$. (9709/32/F/M/23 number 2)

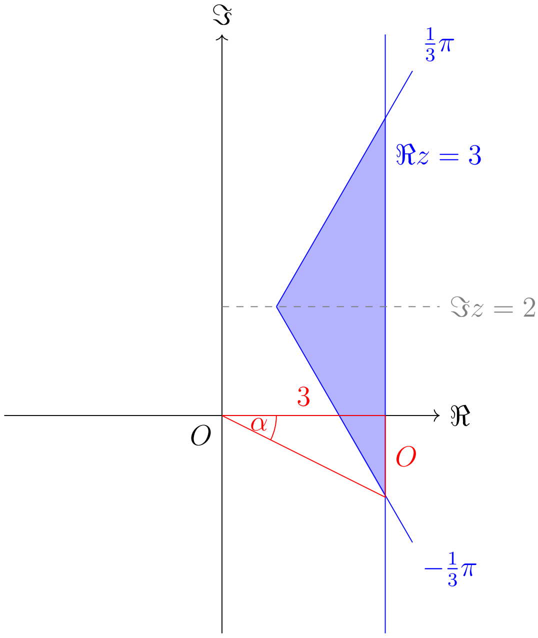
Factor out a negative sign in the first inequality to make the coordinates clear,

$$-\frac{1}{3}\pi \leq \arg(z - (1 + 2i)) \leq \frac{1}{3}\pi$$

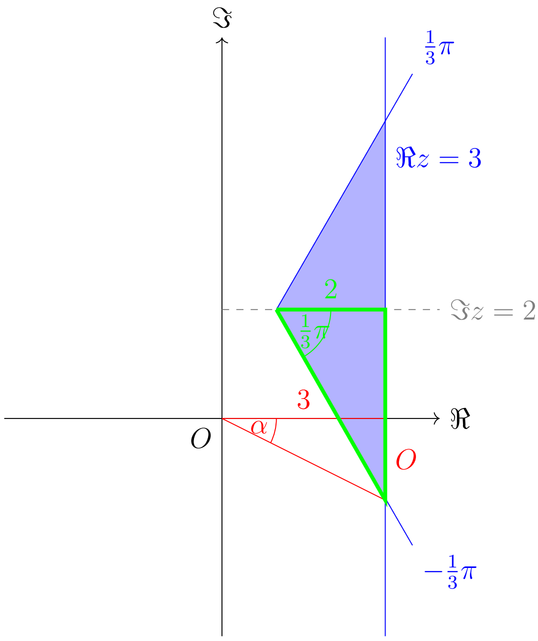
Sketch the argand diagram,



(b) Calculate the least value of $\arg z$ for points in the region from (a). Give your answer in radians correct to 3 decimal places.



α **is the angle we're looking for. But first we need to find the length of the side,** O**, opposite alpha. To do that, let's sketch another triangle,**



This means that to get the length of the opposite side, we can say that,

$$O = 2\tan\left(\frac{1}{3}\pi\right) - 2$$

$$O = 2\sqrt{3} - 2$$

Now let's evaluate α**,**

$$\alpha = \tan^{-1}\left(\frac{2\sqrt{3}-2}{3}\right)$$

$$\alpha = 0.454$$

Our angle is below the x-axis, so this means that the argument will be negative,

$$-0.454$$

Therefore, the final answer is,

$$-0.454$$

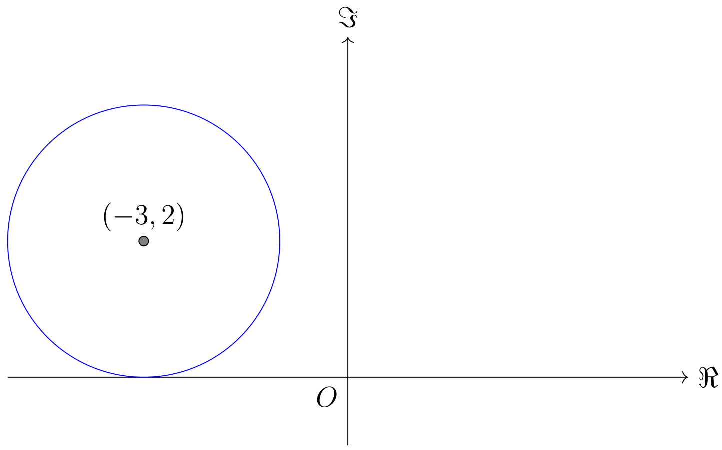
12. (a) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. (9709/32/M/J/23 number 3)

$$|z + 3 - 2i| = 2$$

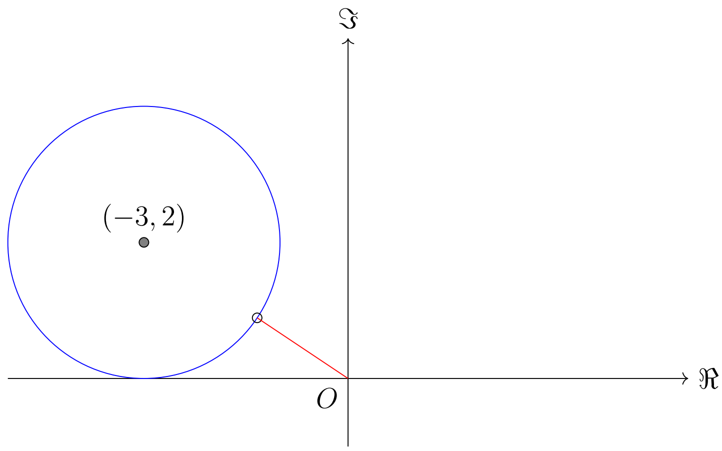
Factor out a negative sign to make the coordinates clear,

$$|z - (-3 + 2i)| = 2$$

Sketch the argand diagram,



(b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form.



We are looking for the length of the red line. We can find the length of this line, by subtracting the radius from the modulus of the complex number $-3+2i$. Let's start by finding the modulus of our complex number,

$$|-3+2i| = \sqrt{3^2+2^2}$$

$$|-3+2i| = \sqrt{13}$$

Now let's subtract the length of the radius from the modulus,

$$\sqrt{13}-2$$

Therefore, the final answer is,

$$\sqrt{13}-2$$