

# Pure 3 May June 2024 Guess Paper

CIE A Level Maths 9709



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## Questions

1. Solve the equation  $2(3^{2x-1}) = 4^{x+1}$ , giving your answer correct to 2 decimal places. [4]
2. (a) On an Argand diagram, shade the region whose points represents the complex numbers  $z$  satisfying the inequalities  $-\frac{1}{3}\pi \leq \arg(z - 1 - 2i) \leq \frac{1}{3}\pi$  and  $\Re z \leq 3$ . [3]  
(b) Calculate the least value of  $\arg z$  for points in the region from (a). Give your answer in radians correct to 3 decimal places. [2]

3. Find, in terms of  $a$ , the set of values of  $x$  satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where  $a$  is a positive constant. [4]

4. (a) Express  $5 \sin \theta + 12 \cos \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]  
(b) Hence solve the equation  $5 \sin 2x + 12 \cos 2x = 6$ , for  $0 \leq x \leq \pi$ . [4]
5. The equation of a curve is  $x^3 + 3x^2y - y^3 = 3$ .  
(a) Show that  $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ . [4]  
(b) Find the coordinates of the points on the curve where the tangent is parallel to the  $x$ -axis. [5]
6. The constant  $a$  is such that  $\int_0^a x e^{-2x} dx = \frac{1}{8}$ . (9709/31/M/J/23 number 9)  
(a) Show that  $a = \frac{1}{2} \ln(4a + 2)$ . [5]  
(b) Verify by calculation that  $a$  lies between 0.5 and 1. [2]  
(c) Use an iterative formula based on the equation in part (a) to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7. Using integration by parts, find the exact value of  $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) dx$ . [5]

8. The variables  $x$  and  $y$  satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for  $x > 0$ . It is given that  $x = 4$  when  $y = 2\sqrt{3}$ .

Solve the differential equation to obtain the value of  $x$  when  $y = 2$ . [8]

9. Solve the equation

$$\frac{5z}{1 + 2i} - zz^* + 30 + 10i = 0$$

giving your answers in the form  $x + iy$ , where  $x$  and  $y$  are real. [5]

10. Relative to the origin  $O$ , the points  $A$ ,  $B$  and  $C$  have position vectors given by

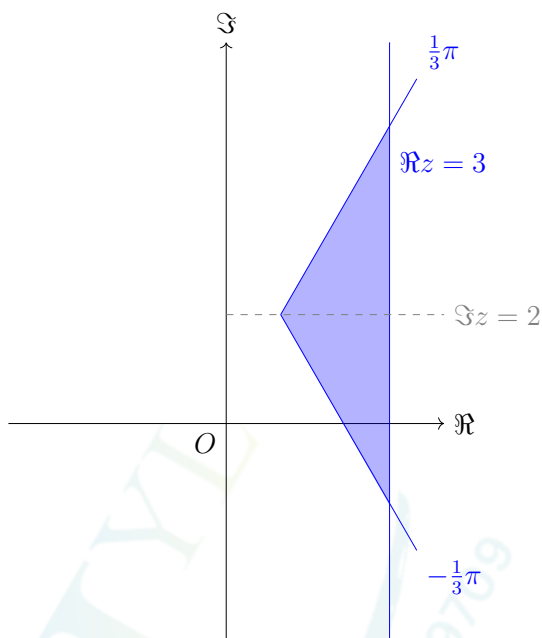
$$\vec{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$$

- (a) Using a scalar product, find the cosine of the angle  $BAC$ . [4]  
(b) Hence find the area of triangle  $ABC$ . Give your answer in a simplified form. [4]
11. Let  $f(x) = \frac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$ .  
(a) Express  $f(x)$  in partial fractions. [5]  
(b) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

## Answers

1.  $x = 2.21$

2. (a)



(b)  $-0.454$

3.  $-\frac{5}{8}a < x < \frac{1}{4}a$

4. (a)  $13 \cos(\theta - 0.395)$

(b)  $x = 0.743$  or  $0.742$ ,  $x = 2.79$

5. (a)  $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$

(b)  $(-2, 1)$ ,  $(0, -\sqrt[3]{3})$

6. (a)  $a = \frac{1}{2} \ln(4a + 2)$

(b) Justify the given statement

(c)  $0.84$

7.  $\frac{1}{2}\pi - \ln 2$

8.  $\frac{1}{2} \ln(y^2 + 4) + 2 \tan^{-1} \frac{y}{2} = \ln x + \frac{2\pi}{3}$

9.  $3 - 4i$   $6 + 2i$

10. (a)  $\frac{1}{3}$

(b)  $5\sqrt{2}$

11. (a)  $\frac{2}{1+2x} + \frac{2}{3-x} - \frac{3}{(3-x)^2}$

(b)  $\frac{7}{3} - 4x + \frac{215}{27}x^2$