Pure 1 October November 2024
Marking Scheme

CIE A Level Maths 9709

GCEALevelMaths9709

Question	Answer	Marks	Guidance
1	$[\,f(x)=\,]\,\frac{\frac{4}{3}x^{\frac{1}{3}}}{\frac{1}{3}}-\frac{2x^{\frac{2}{3}}}{\frac{2}{3}}\ [+c]$	**B1 B1**	$\frac{1}{3}$ and $\frac{2}{3}$ may be seen as sums of 1 and a fraction.
	$8=4-3+c$	**M1**	Substituting $(1,8)$ into an integral.
	$[\,f(x)=\,]4x^{\frac{1}{3}}-3x^{\frac{2}{3}}+7$	**A1**	Fractions in the denominator scores A0.
		4	

Question	Answer	Marks	Guidance
2(a)	$5C2\times\left[x^2\right]^3\times\left[\frac{a}{x}\right]^2\quad,\quad 5C3\times\left[x^2\right]^2\times\left[\frac{a}{x}\right]^3$	**B1 B1**	SOI can be seen in expansion
	$2\left(10\times a^2\right)=10\times a^3$	**M1**	SOI terms must be from a correct series
	$a=\frac{2(10)}{10}=2$	**A1**	
		4	
2(b)	-40	**B1**	
		1	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \{\frac{1}{4}(2x-3)\times 2\} \times \{2\}$	**B1 B1**	May see $\frac{1}{4}(8x-12)$
	$2x-3=1$	**M1**	Equate their $\frac{dy}{dx}$ to 1
	$x=2$	**A1**	
		4	

Question	Answer	Marks	Guidance
4	{Stretch} {factor 2} {in y-direction}	**B2,1,0**	2 out of 3 scores B1
	{Translation} $\begin{pmatrix} \{0\} \\ \{3\} \end{pmatrix}$	**B2,1,0**	Accept shift
		4	

Question	Answer	Marks	Guidance
5(a)	$1+1-a+b+16=0\,[\Rightarrow a-b=18]$ $16-4a+16=0\,[\Rightarrow 4a=32]$	**B1 B1**	B1 for each equation. Allow unsimplified. Can be implied by correct values for a and b
	$a=8,\ b=-10$	**B1**	
	Centre is $\left(-\frac{their\ a}{2}, -\frac{their\ b}{2}\right)$ $[-4,5]$	**B1 F1**	Or $x=-4$, $y=5$
		4	

Question	Answer	Marks	Guidance
5(b)	Gradient of AC is $\frac{1-their\ y}{1-their\ x}$ $[= \frac{1-5}{-1--4} = \frac{1-5}{-1+4} = -\frac{4}{3}]$	*M1	Using *their* centre correctly
	Gradient of tangent is $= \frac{-1}{their\ -\frac{4}{3}}$ $[= \frac{3}{4}]$	A1 F1	Use of $m_1m_2 = -1$ to obtain the gradient of the tangent
	Equation: $y-1 =' their\ \frac{3}{4}'(x+1)$ or $y = \frac{3}{4}x+\frac{7}{4}$	DM1	Using $(-1,1)$ with *their* gradient of the tangent at A
	$3x-4y=-7$ or $-4y+3x=-7$. or integer multiples of these	A1	
	Alternative method for question 5(b)		
	$2x+2y\frac{dy}{dx}+8-10\frac{dy}{dx}=0$	*M1	Implicit differentiation with at least one y term differentiated correctly
	$8\frac{dy}{dx}=6 \Rightarrow \frac{dy}{dx}=\frac{6}{8}$	A1	
	Equation: $y-1 =' their\ \frac{3}{4}'(x+1)$ or $y = \frac{3}{4}x+\frac{7}{4}$	DM1	Using $(-1,1)$ with *their* gradient of the tangent at A
	$3x-4y=-7$ or $-4y+3x=-7$. or integer multiples of these	A1	
	Alternative method for question 5(b)		
	$\frac{dy}{dx} = -\frac{1}{2}\left(25-(x+4)^2\right)^{-\frac{1}{2}}(-2x-4)$	*M1	Rearranging to form $y=$ and differentiating using the chain rule
	$\frac{dy}{dx} = -\frac{1}{2}(25-9)^{\frac{1}{2}}(-6) = \frac{6}{8}$	A1	
	Equation: $y-1 =' their\ \frac{3}{4}'(x+1)$ or $y = \frac{3}{4}x+\frac{7}{4}$	DM1	Using $(-1,1)$ with *their* gradient of the tangent at A

Question	Answer	Marks	Guidance
5(b)	$3x-4y=-7$ or $-4y+3x=-7$. or integer multiples of these	A1	
		4	

Question	Answer	Marks	Guidance
6(a)	Use identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$	M1	
	Use identity $\cos^2\theta = 1-\sin^2\theta$	M1	
	$\pm\left(6\sin^2\theta - 5\sin\theta - 4 = 0\right)$	A1	
		3	
6(b)	Attempt solution of *their* 3 term equation and correct process to find at least 1 value of $\sin x$ or $\sin 2x$ or $\sin\theta$	M1	Expect $(3s-4)(2s+1)=0$, $s=-\frac{1}{2}$
	$x=105°$	A1	Or greater accuracy **B1 SC** if no solution to the quadratic
	$x=165°$ or $(90+(180-\textit{their}\ 105))$; and no other solutions for $0°<x<180°$	A1 FT	WWW **B1 SC** FT if no solution to the quadratic **B1 SC** both correct in radians, 1.85, 2.88
		3	

Question	Answer	Marks	Guidance
7(a)	**Either** Let midpoint of PS be H: $\sin HCP = \frac{3}{6} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$ **Or** $\sin PQS = \frac{6}{12} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$ **Or** using cosine rule: angle $PCS = \frac{\pi}{3}$ **Or** by inspection: triangle PCS is equilateral so angle $PCS = \frac{\pi}{3}$	**M1**	
	Angle $PCQ = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	**A1**	AG
		2	
7(b)	Perimeter $= 2 \times 6 \times \frac{2}{3}\pi$ or $12\pi - 4\pi$	**M1**	Length of two arcs PQ and RS
	$+2 \times 3 \times \pi$	**M1**	Adding circumference of two semicircles
	14π	**A1**	Must be a single term
		3	
7(c)	Area sector $CPS = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$	**M1**	Uses correct formula for sector
	Area of large segment of large circle beyond CPS $= 6\pi - \frac{1}{2} \times 6^2 \times \sin\left(\frac{\pi}{3}\right) = 6\pi - 9\sqrt{3}$	**M1**	Attempts to find area of segment
	Area of small semicircle $= \frac{1}{2} \times \pi \times 3^2$ or area of small circle $= \pi \times 3^2$	**M1**	
	Area of plate $=$ Large circle $-[2\times]$ small semicircle $-[2\times]$ segment area	**M1**	
	$\pi \times 6^2 - \pi \times 3^2 - 2 \times \left(6\pi - 9\sqrt{3}\right) = 15\pi + 18\sqrt{3}$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
7(c)	**Alternative method for question 7(c)**		
	Area of sector $PCQ = \frac{1}{2} \times 6^2 \times \frac{2\pi}{3} = 12\pi$	**M1**	Uses correct formula for sector
	Area of triangle $PCS = \frac{1}{2} \times 6^2 \times \sin\frac{\pi}{3} = 9\sqrt{3}$	**M1**	Uses correct formula for triangle
	Area of small semicircle $= \frac{1}{2} \times \pi \times 3$ or area of circle $= \pi \times 3^2$	**M1**	
	Area of plate $= [2\times]$ large sector $+[2\times]$ triangle $-[2\times]$ small semicircle	**M1**	
	$2(12\pi) + 2(9\sqrt{3}) - \pi \times 3^2 = 15\pi + 18\sqrt{3}$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
8	$(x-2)^2 + (y+1)^2 = 20$ $y = mx + 13$ leading to $(x-2)^2 + (mx+14)^2 = 20$	**M1**	Finding equation of tangent and substituting into circle equation. Must be $mx + 13$
	$x^2 - 4x + 4 + m^2x^2 + 28mx + 196 = 20$ leading to $\left(m^2+1\right)x^2 + (28m-4)x + 180[=0]$	**M1**	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant. m cannot be numeric.
	$(28m-4)^2 - 4\left(m^2+1\right) \times 180[=0]$	***M1**	Use of $b^2 - 4ac$. Not in quadratic formula. m cannot be numeric, c must be numeric.
	$64m^2 - 224m - 704[=0]$ leading to $\left[2m^2 - 7m - 22 = 0\right]$	**DM1**	Simplifies to 3 term quadratic.
	$m = \frac{11}{2}$ or $m = -2$	**A1**	Condone no method for solving quadratic shown

Question	Answer	Marks	Guidance
8	$m = \frac{11}{2}$ leading to $125x^2 + 600x + 720 = 0$ leading to $x = -\frac{12}{5}$	DM1	Must be correct x for *their* quadratic
	$m = -2$ leading to $5x^2 - 60x + 180 = 0$ leading to $x = 6$	DM1	Must be correct x for *their* quadratic
	$\left(-\frac{12}{5}, \frac{1}{5}\right)$, $(6, 1)$	A1	
	Alternative Method 1 for first 4 marks of Question 8		
	$\frac{\lvert m(2)-(-1)+13\rvert}{\sqrt{m^2+1}}$	(M1)	Use of formula for the length of a perpendicular from a point to a line
	$\frac{\lvert m(2)-(-1)+13\rvert}{\sqrt{m^2+1}} = \sqrt{20}$	(M1)	Equates length of a perpendicular from a point to a line to the radius
	$(2m+14)^2 = 20\left(m^2+1\right)$	(M1)	Squares and clears the fraction
	$16m^2 - 56m - 176 = 0$ [leading to $4m^2 - 14m - 44 = 0$]	(M1)	
		8	

Question	Answer	Marks	Guidance
9(a)	$a\left(x + \frac{4}{x}\right) + 1$	B1	ISW
		1	
9(b)	$a\left(1 + \frac{4}{1}\right) + 1 = 26$	M1	Substitute $x = 1$ into *their* expression from **(a)** and equate to 26 This may be done in 2 stages: $f(1) = 5$, $g(5) = 26$
	$[a =]5$	A1	
		2	

Question	Answer	Marks	Guidance
9(c)	No, [because it is] not one-one	**B1**	Or other suitable explanation that may include one to many or many to one
		1	
9(d)	$[g^{-1}(x)] = \frac{x-1}{3}$ WWW	**B1**	Condone use of a instead of 3
	$[g^{-1}f(x)] = \frac{x+\frac{4}{x}-1}{3}$ OE	**M1**	Correct combination of their $g^{-1}(x)$ with given $f(x)$ Condone use of a instead of 3
	$[g^{-1}f(x)] = \frac{x^2-x+4}{3x}$ or $\frac{1}{3}\left(x+\frac{4}{x}-1\right)$ or $\frac{1}{3}\left(x+4x^{-1}-1\right)$ OE ISW	**A1**	Must not contain unresolved fractions e.g $\frac{x+4x^{-1}-1}{3}$
		3	
9(e)	The domain of f does not include the whole range of g. **Or** The range of f does not lie in the domain of f	**B1**	Accept an answer that includes an example outside of the domain of f, e.g. $g(-1) = -2$ but for f, $x > 0$
		1	

Question	Answer	Marks	Guidance
10(a)	$2(2p+3) = 17+31-3p$	***M1**	
	$p = 6$	**A1**	
	$d = (2(6)+3)-17[=-2]$	**DM1**	Using their p to find d

Question	Answer	Marks	Guidance
10(a)	8^{th} term $= 17 + 7d = 3$	A1	
	Alternative Method for first 3 marks of Question 10(a)		
	$d = 2p - 14$, $d = 28 - 5p$	(*M1)	Allow unsimplified or equivalent
	Solving simultaneously to find p or d	(DM1)	
	$[p = 6]$, $d = -2$	(A1)	
		4	
10(b)	$(5q-8)^2 = 16(13-q) \Rightarrow 25q^2 - 64q - 144[=0]$	M1	
	$(25q+36)(q-4)[=0]$ leading to $[\Rightarrow q = 4]$	M1	Solve 3-term quadratic with real solutions
	$[r =]\frac{3}{4}$	A1	Ignore $-\frac{19}{20}$
	$S_\infty = \frac{16}{1-\frac{3}{4}} = 64$	A1	Ignore extra solution **SC B1** if no method shown for solving quadratic
	Alternative Method for Question 10(b)		
	$16r = 5q - 8$, $16r^2 = 13 - q$ leading to $80r^2 + 16r - 57 = 0$	(M1)	
	$(4r-3)(20r+19) = 0$	(M1)	Solve 3-term quadratic with real solutions
	$r = \frac{3}{4}$	(A1)	Ignore $\frac{-19}{20}$
	$S_\infty = \frac{16}{1-\frac{3}{4}} = 64$	A1	Ignore extra solution **SC B1** if no method shown for solving quadratic
		4	

Question	Answer	Marks	Guidance
11(a)	$[\pi]\int \frac{16}{(x-2)^4}\,[dx] = [\pi]\int 16(x-2)^{-4}[dx] = [\pi]\left(-\frac{16}{3\times(x-2)^3}\right)$	*M1	Integrate y^2 (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{3}x(x-1)^{-3}$
	$[\pi]\left(-\frac{16}{3\times(x-2)^3}\right)$	A1	OE
	$[\pi]\left(-\frac{16}{3\times 8}+\frac{16}{3\times 1}\right)\ \left[=[\pi]\frac{14}{3}\right]$	DM1	Sub correct limits into *their* integral: $F(4)-F(3)$ Must see at least $\left(-\frac{2}{3}+\frac{16}{3}\right)$. Allow 1 sign error. Decimal: 4.66π or 14.66
	Volume of cylinder $\left[=\pi\times 1^2\times 1\right]=\pi$ OR $[\pi]\int_3^4 1\,[dx]=\pi$	B1	π or $\pm\pi(4-3)$ seen.
	Volume of revolution $\left[=\frac{14}{3}\pi-\pi\right]=\frac{11}{3}\pi$	A1	A0 for 11.5 (not exact) If DM0 for insufficient substitution, or B0, SC B1 for $\frac{11}{3}\pi$
		5	
11(b)	$\left[\frac{dy}{dx}=\right]\left\{-8(x-2)^{-3}\right\}$	B2	OE
	At B gradient $=-1$	B1	
	Eqn of tangent $y-1=$ *their* "-1"$(x-4)$ OR Eqn of normal $y-1=$ *their* "1"$(x-4)$	M1	SOI Following differentiation OE e.g. $y=-x+5$ or $y=x-3$. (Must have $m_N=-\frac{1}{m_T} forM1$)
	Tangent crosses x-axis at 5 **or** normal crosses x-axis at 3	A1	SOI For at least one intercept correct or correct to integration.
	Area $=1$	A1	From lengths: $\frac{1}{2}\times 2\times 1=1$ or by integration
		6	