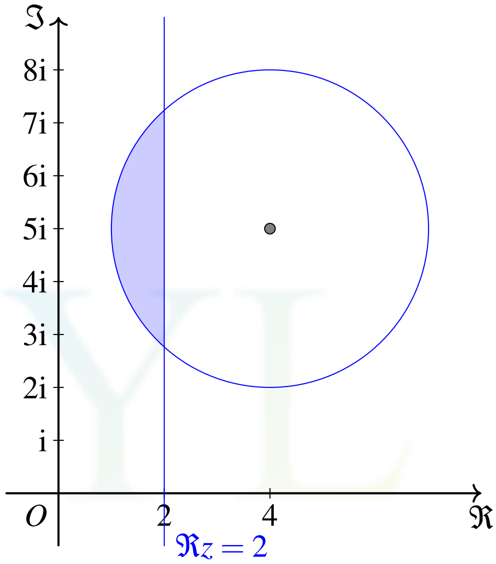
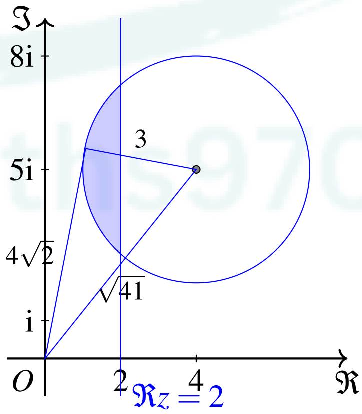
Pure 3 October November 2024
Marking Scheme

CIE A Level Maths 9709

*This is not endorsed by Cambridge and is purely for practice purposes only.

Question	Answer	Marks	Guidance
1	Use laws of indices correctly and solve for 4^x	**M1**	
	Obtain correct solution in any form, e.g $4^x = \frac{64}{63}$	**A1**	
	Use a correct method for solving en equation of the form $4^x = a$, where $a > 0$	**M1**	
	Obtain answer 0.0114	**A1**	
		4	

Question	Answer	Marks	Guidance
2(a)	Show a circle with centre $4+5i$. Accept curved shape with correct point roughly in the middle.	**B1**	
	Show a circle with radius 3 and centre not at the origin. The shape should be consistent with their scales.	**B1**	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	**B1**	
	Shade the correct region on a correct diagram. Any other shading must be accompanied with words to explain which region is required.	**B1**	
		4	
2(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\sin^{-1}\left(\frac{3}{\sqrt{41}}\right)+\tan^{-1}\left(\frac{5}{4}\right)$ $(0.4876+0.8961)$	**M1**	
	Obtain answer 1.38 radians or 79.3°	**A1**	
		2	

Question	Answer	Marks	Guidance
3	Use the correct product rule and then the chain rule to differentiate either $\sin^3 x$ or $\sqrt{\cos x}$	**M1**	e.g two terms with one part of $\frac{dy}{dx} = p\sin^2 x\cos x\sqrt{\cos x} + q\frac{\sin^3 x\sin x}{\sqrt{\cos x}}$
	Obtain correct derivative in any form e.g $3\sin^2 x\cos x\sqrt{\cos x} - \frac{\sin^3 x\sin x}{2\sqrt{\cos x}}$	**A1 A1**	A1 for each correct term substituted in the complete derivative
	Equate their derivative to zero and obtain a horizontal equation with positive integer powers of $\sin x$ and/or $\cos x$ from an equation including $\sqrt{\cos x}$ or $\frac{1}{\sqrt{\cos x}}$ using sensible algebra.	**M1**	e.g. $3\sin^2 x\cos^2 x - \frac{1}{2}\sin^4 x = 0$
	Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	**M1**	Can be awarded before the previous M1. May involve more than one trigonometric term.
	Obtain $7\cos^2 x = 1$, $7\sin^2 x = 6$ or $\tan^2 x = 6$, or equivalent, and obtain answer $x = 1.18$	**A1**	CAO. The question asks for 3sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. 67.6° scores A0.
		6	

Question	Answer	Marks	Guidance
4	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan x$ or an equation in $\cos x$ and $\sin x$	**M1**	e.g $\frac{\tan x+\tan 30^\circ}{1-\tan x\tan 30^\circ}=\frac{3}{\tan x}$ Allow if 3 in denominator or $\frac{\sin x\cos 30^\circ+\cos x\sin 30^\circ}{\cos x\cos 30^\circ-\sin x\sin 30^\circ}=\frac{3\cos x}{\sin x}$.
	Obtain correct 3 term equation $3\tan^2 x+4\sqrt{3}\tan x-9=0$, or equivalent	**A1**	or $4\sin x\cos x=3\sqrt{3}\cos^2 x-\sqrt{3}\sin^2 x$
	Solve a 3-term quadratic in $\tan x$ and obtain a value for x	**M1**	
	Obtain answer, e.g. 42.8°	**A1**	42.829...
	Obtain second answer, e.g. 107.2° and no other	**A1**	107.170... Ignore answers outside the given interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
5	State $\frac{dy}{d\theta} = 3\cos\theta - 1$	**B1**	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\frac{dy}{dx}$ for final A1
	Use correct quotient rule, or product rule if rewrite x as $\sin\theta(3-\cos\theta)^{-1}$	**M1**	Incorrect formula seen M0 A0 otherwise BOD.
	Obtain $\frac{dx}{d\theta} = \frac{(3-\cos\theta)\cos\theta - \sin^2\theta}{(3-\cos\theta)^2}$ o.e	**A1**	$\cos\theta(3 - \cos\theta)^{-1} - \sin\theta(3 - \cos\theta)^{-2}(\sin\theta)$ or equivalent
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	$\left(\frac{dy}{dx} = (3\cos\theta - 1) \div \frac{3\cos\theta - 1}{(3-\cos\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
	Obtain $\frac{dy}{dx} = (3-\cos\theta)^2$.	**A1**	AG - must see working in above cell to gain final A1. Allow $\cos^2\theta + \sin^2\theta = 1$ to be implied. x instead of θ or missing θ more than twice on the right side then A0 final mark.
		5	

Question	Answer	Marks	Guidance
6	State that $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{1}{2\sqrt{x}}dx$	**B1**	
	Substitute throughout for x and dx	**M1**	
	Obtain a correct integral with integrand $\frac{4}{u^2+4}$	**A1**	
	Integrate and obtain term of the form $a\tan^{-1}\frac{u}{b}$	**M1**	$\left(2\tan^{-1}\left(\frac{u}{2}\right)\right)$
	Use limits 2 and ∞ for u or equivalent and evaluate trig	**A1**	e.g. $2\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$ Must be working in radians.
	Obtain answer $\frac{1}{2}\pi$	**A1**	Or equivalent single term
		6	

Question	Answer	Marks	Guidance
7	Solve for v or w	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain $v = \frac{-11+18i}{5+8i}$ or $w = \frac{-14+31i}{5+8i}$	**A1**	
	Multiply numerator and denominator by the conjugate of the denominator	**M1**	
	Obtain $v = 1+2i$	**A1**	
	Obtain $w = 2+3i$	**A1**	
		6	

Question	Answer	Marks	Guidance
8(a)	Commence integration and reach $a\sqrt{x}\ln x^2 + b\int \frac{1}{x} \times \sqrt{x}\ dx$ or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x^2 - \int \frac{2}{x} \times 2\sqrt{x}\,dx$, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x^2 - 8\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 4	DM1	
	Rearrange and obtain $a = \sqrt{exp\left(4 - \frac{2}{\sqrt{a}}\right)}$	A1	Obtain **given answer** from full and correct working
		5	
8(b)	Calculate the values of a relevant expression or pair of expressions at $a = 4$ and $a = 5$	M1	
	Complete the argument correctly with correct values	A1	
		2	
8(c)	Use the iterative process $a_{n+1} = \sqrt{exp\left(4 - \frac{2}{\sqrt{a_n}}\right)}$ correctly at least once	M1	
	Obtain answer 4.64	A1	
	Show sufficient iterations to 4 dp to justify 4.64 to 2 dp, or show there is a sign change in the interval $(4.635, 4.645)$	A1	e.g $4, 4.4817, 4.6073, 4.6372, 4.6442, \ldots$
		3	

Question	Answer	Marks	Guidance
9(a)	Show sufficient working to justify the given statement	B1	e.g see $2\csc\theta \times -\csc\theta\cot\theta$ in the working or express in terms of $\sin\theta$ and $\cos\theta$ and use quotient rule to obtain the given result. Solution must have θ present throughout and must reach the given answer.
		1	
9(b)	Separate variables correctly Check for relevant working in **(a)**	B1	$\int 4x\,dx = \int \frac{3\sec^2\theta}{\sin^2\theta} - \frac{3}{\sin^2\theta} - \frac{2\cot\theta}{\sin^2\theta}\,d\theta$ condone incorrect notation e.g. missing dx. Need either the integral sign or the dx, dθ.
	Obtain term $2x^2$	B1	
	Obtain terms $3\tan\theta + \csc^2\theta$	B1 + B1	
	Form an equation for the constant of integration, or use limits $x = 1$, $\theta = \frac{1}{4}\pi$, in a solution with **at least two correctly obtained terms** of the form ax^2, $b\tan\theta$ and $c\csc^2\theta$, where $abc \neq 0$	M1	Need to have 3 terms. Constant of correct form.
	State correct solution in any form, e.g. $2x^2 + 3 = 3\tan\theta + \csc^2\theta$	A1	
	Substitute $\theta = \frac{1}{6}\pi$ and obtain answer $x = 1.17$	A1	1.1687... $\sqrt{\frac{1+\sqrt{3}}{2}}$ If see a correctly rounded value ISW
		7	

Question	Answer	Marks	Guidance
10(a)	Carry out correct method for finding a vector equation for AB	M1	
	Obtain $[\mathbf{r} =]\mathbf{i}+2\mathbf{j}+3\mathbf{k}+t(\mathbf{i}-2\mathbf{j}+\mathbf{k})$	A1	OE e.g. $\mathbf{r} = 2\mathbf{i}+4\mathbf{k}+t(\mathbf{i}-2\mathbf{j}+\mathbf{k})$
	Equate two pairs of components of general points on their AB and l and evaluate s and t	M1	$\begin{pmatrix} 1+t \\ 2-2t \\ 3+t \end{pmatrix} = \begin{pmatrix} -1-2s \\ 4+3s \\ 7-4s \end{pmatrix}$
	Obtain correct answer for s or t, e.g. $s = -2, t = 2$	A1	Correct value from two correct component equations.
	Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $5 \neq 15$)	A1	Conclusion needs to follow correct values.
		5	
10(b)	Find $\overline{AP}$ for a general point P on l e.g. $-2\mathbf{i}+2\mathbf{j}+4\mathbf{k}+s(-2\mathbf{i}+3\mathbf{j}-4\mathbf{k})$	B1	OE
	Calculate scalar product of *their* $\overline{AP}$ and a direction vector for l and equate the result to zero	M1	e.g. $4+4s+6+9s-16+16s = 0$ M0 if using $\overline{OP}$ M0 if using parallel line through A
	Obtain $s = \frac{6}{29}$	A1	
	Obtain answer $-\frac{41}{29}\mathbf{i}+\frac{134}{29}\mathbf{j}+\frac{179}{29}\mathbf{k}$	A1	Accept coordinates in place of position vector.
		4	

Question	Answer	Marks	Guidance
11(a)	State or imply the form $\frac{A}{2+x}+\frac{B}{1+x}+\frac{C}{(1+x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A=-6$, $B=8$ and $C=-3$	**A1**	SR after B0 can score M1A1 for one correct value
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
11(b)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(1+x)^{-1}$, $(1+x)^{-2}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**	$-3\left(1-\frac{1}{2}x+\frac{1}{4}x^2\right)$ $8\left(1-x+x^2\right)$ $-3\left(1-2x+3x^2\right)$
	Obtain final answer $2-\frac{1}{2}x-\frac{7}{4}x^2$	**A1**	
		5	